1. VISION, MISSION, PROGRAM EDUCATIONAL OBJECTIVES

(A) VISION

To become a renowned department imparting both technical and non-technical skills to the students by implementing new engineering pedagogy’s and research to produce competent new age electrical engineers.

(B) MISSION

- To transform the students into motivated and knowledgeable new age electrical engineers.
- To advance the quality of education to produce world class technocrats with an ability to adapt to the academically challenging environment.
- To provide a progressive environment for learning through organized teaching methodologies, contemporary curriculum and research in the thrust areas of electrical engineering.

(C) PROGRAM EDUCATIONAL OBJECTIVES

PEO 1: Apply knowledge and skills to provide solutions to Electrical and Electronics Engineering problems in industry and governmental organizations or to enhance student learning in educational institutions

PEO 2: Work as a team with a sense of ethics and professionalism, and communicate effectively to manage cross-cultural and multidisciplinary teams

PEO 3: Update their knowledge continuously through lifelong learning that contributes to personal, global and organizational growth

(D) PROGRAM OUTCOMES

PO 1: Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals and an engineering specialization to the solution of complex engineering problems.

PO 2: Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural science and engineering sciences.

PO 3: Design/development of solutions: design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal and environmental considerations.
PO 4: Conduct investigations of complex problems: use research based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

PO 5: Modern tool usage: create, select and apply appropriate techniques, resources and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

PO 6: The engineer and society: apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

PO 7: Environment sustainability: understand the impact of the professional engineering solutions in the societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

PO 8: Ethics: apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

PO 9: Individual and team work: function effectively as an individual and as a member or leader in diverse teams, and in multidisciplinary settings.

PO 10: Communication: communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

PO 11: Project management and finance: demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

PO 12: Lifelong learning: recognize the need for, and have the preparation and ability to engage in independent and lifelong learning in the broader context of technological change.
(E) PROGRAM SPECIFIC OUTCOMES

**PSO-1:** Students will establish themselves as effective professionals by solving real problems through the use of computer science knowledge and with attention to teamwork, effective communication, critical thinking and problem solving skills.

**PSO-2:** Students will demonstrate their ability to adapt to a rapidly changing environment by having learned and applied new skills and new technologies.

**PSO-3:** Students will be provided with an educational foundation that prepares them for excellence, leadership roles along diverse career paths with encouragement to professional ethics and active participation needed for a successful career.
2. SYLLABUS (UNIVERSITY COPY)

II Year – I SEMESTER

ELECTROMAGNETIC FIELDS

Preamble:

Electromagnetic fields are the pre-requisite for most of the subjects in the gamut of electrical engineering. The study of this subject enables students to understand and interpret the phenomenon pertinent to electrical engineering using microscopic quantities such as electric and magnetic field intensities, scalar and vector potentials.

Learning objectives:

- To study the production of electric field and potentials due to different configurations of static charges.
- To study the properties of conductors and dielectrics, calculate the capacitive of various configurations and understand the concept of conduction and convection current densities.
- To study the magnetic fields produced by currents in different configurations, application of ampere’s law and the Maxwell’s second and third equations.
- To study the magnetic force and torque through Lorentz force equation in magnetic field environment like conductors and other current loops.
- To develop the concept of self and mutual inductances and the energy stored.
- To study time varying and Maxwell’s equations in different forms and Maxwell’s fourth equation for the induced e.m.f.

UNIT – I Electrostatics:

Electrostatic Fields – Coulomb’s Law – Electric Field Intensity (EFI) – EFI due to a line and a surface charge – Work done in moving a point charge in an electrostatic field – Electric Potential – Properties of potential function – Potential gradient – Guass’s law — Maxwell’s first law, div(\( \mathbf{D} \)) = \( \rho \) Laplace’s and Poisson’s equations and Solution of Laplace’s equation in one variable.

UNIT – II Conductors – Dielectrics and Capacitance:

Electric dipole – Dipole moment – potential and EFI due to an electric dipole – Torque on an Electric dipole in an electric field – Behaviour of conductors in an electric field – Conductors and Insulators

Polarization – Boundary conditions between conduction to Dielectric and dielectric to dielectrics capacitance – capacitance of parallel plates, spherical and coaxial cables with composite dielectrics – Energy stored and energy density in a static electric field – Current density – conduction and Convection current densities – Ohm’s law in point form – Equation of continuity

UNIT – III Magneto statics and Ampere’s Law:

Static magnetic fields – Biot-Savart’s law – Oesterd’s experiment - Magnetic field intensity (MFI) – MFI due to a straight current carrying filament – MFI due to circular, square and solenoid current – Carrying wire – Relation between magnetic flux, magnetic flux density and MFI – Maxwell’s second Equation, \( \text{div}(\mathbf{B})=0 \) – Ampere’s circuital law and its applications viz. MFI due to an infinite sheet of current and a long filament carrying conductor – Point form of Ampere’s circuital law – Field due to a circular loop, rectangular and square loops, Maxwell’s third equation, \( \text{Curl}(\mathbf{H})=\mathbf{J} \).
UNIT – IV  Force in Magnetic fields:
Magnetic force - Moving charges in a Magnetic field – Lorentz force equation – force on a current element in a magnetic field – Force on a straight and a long current carrying conductor in a magnetic field – Force between two straight long and parallel current carrying conductors – Magnetic dipole and dipole moment – a differential current loop as a magnetic dipole – Torque on a current loop placed in a magnetic field.

UNIT – V  Self and Mutual inductance:
Self and Mutual inductance – determination of self-inductance of a solenoid and toroid and mutual inductance between a straight long wire and a square loop wire in the same plane – energy stored and density in a magnetic field.

UNIT – VI  Time Varying Fields:

Learning outcomes:
- To Determine electric fields and potentials using gauss’s law or solving Laplace’s or Poisson’s equations, for various electric charge distributions.
- To Calculate and design capacitance, energy stored in dielectrics.
- To Calculate the magnetic field intensity due to current, the application of ampere’s law and the Maxwell’s second and third equations.
- To determine the magnetic forces and torque produced by currents in magnetic field
- To determine self and mutual inductances and the energy stored in the magnetic field.
- To calculate induced e.m.f., understand the concepts of displacement current and Poynting vector.

Text Books:

Reference Books:
3. COURSE OBJECTIVES AND COURSE OUTCOMES

(a) COURSE OBJECTIVES

(a) Outline the concepts of electric field, magnetic field
(b) Apply the concept of electric and magnetic fields in machines
(c) Get solid foundation in engineering.
(d) Solve problems and also to pursue Higher studies.

(b) COURSE OUTCOMES

(CO1) Compute the force, fields & Energy for different charge & current Configurations.
(CO2) Evaluate capacitance and inductance
(CO3) Analyze Maxwell’s equation in different forms (Differential and integral)
(CO4) Analyze Lorentz force equations and self and mutual inductances
(CO5) Analyze time varying fields of Electro- Magnetic fields to understand transmission lines

(c) TOPIC OUTCOMES

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<tr>
<th>S.N.</th>
<th>TOPIC</th>
<th>TOPIC OUTCOMES</th>
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<td><strong>UNIT – I</strong></td>
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<td>1</td>
<td>Introduction to Electromagnetic fields</td>
<td>Explain nature of electromagnetic fields</td>
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<td>Coulombs law</td>
<td>Derive coulombs law and calculate force in various cases</td>
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<td>3</td>
<td>Electrical field intensity</td>
<td>Apply and calculate Electrical field intensity due to various conditions</td>
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<td>point charge in an electrostatic field</td>
<td>Derive Work done in moving a point charge in an electrostatic field</td>
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<td>Electric Potential</td>
<td>Calculate Electric field and Potential</td>
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<td>potential function – Potential</td>
<td>Explain Properties of potential function</td>
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<tr>
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<td>gradient</td>
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<tr>
<td>8</td>
<td>Gauss’s Law</td>
<td>Application of Gauss’s Law to complex electrical field calculations</td>
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<td>9</td>
<td>Maxwell’s first law</td>
<td>Analyze the Maxwell’s first law and calculation of Electric Potential</td>
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<td>Laplace’s equation in one variable</td>
<td>Derive Laplace’s equation to 3 variable</td>
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<td>Electric dipole</td>
<td>Calculate the Dipole and dipole moment</td>
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<td>12</td>
<td>Torque of Electric dipole</td>
<td>Apply Electric dipole in an molecule</td>
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<td>13</td>
<td>Conductors and Insulators</td>
<td>Explain the behavior of conductors in an electric field</td>
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<tr>
<td>14</td>
<td>Problems on Gauss’s Law</td>
<td>Explain Gauss’s Law to complex electrical field calculations</td>
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<td>Problems on Laplace’s equation</td>
<td>Derive Laplace’s equation</td>
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<td>16</td>
<td>Problems on Electrical field intensity</td>
<td>Apply of Columbus law and Electrical field</td>
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<td>17</td>
<td>Laplace’s equation of 2 variable</td>
<td>Apply of Laplace’s equation in real time</td>
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**UNIT - II**

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<tr>
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<td>Dielectrics &amp; Capacitance</td>
<td>Explain dielectrics &amp; capacitance</td>
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<tr>
<td>18</td>
<td>conductors in an electric field</td>
<td>Analyze behavior of dielectrics &amp; capacitance in an electric field</td>
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<tr>
<td>19</td>
<td>field inside a dielectric material</td>
<td>Identify physically dielectric properties</td>
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<td>20</td>
<td>Dielectric – Conducto</td>
<td>Explain the polarization – dielectric – conductor and dielectric</td>
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<tr>
<td>21</td>
<td>Dielectric boundary conditions</td>
<td>Solve dielectric boundary conditions And Describe dielectric boundary conditions</td>
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<td>composite dielectrics-1</td>
<td>Analyze the phenomenon of Capacitance – Capacitance of parallel plots– spherical co-axial capacitors</td>
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<td>23</td>
<td>composite dielectrics-2</td>
<td>Derive composite dielectrics boundary</td>
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<td>energy density in a static electric field</td>
<td>Explain the energy stored and energy density in a static electric field</td>
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<td>25</td>
<td>Conduction and Convection current densities</td>
<td>Apply Ohm’s Law in point forms Recognize current density</td>
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<td>26</td>
<td>Static magnetic fields</td>
<td>Explain the static magnetic fields and Observe the static magnetic field</td>
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<tr>
<td>27</td>
<td>Biot-Savart’s law</td>
<td>Apply the biot - Savart’s law and calculate magnetic field strength</td>
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<td>Derive the magnetic field intensity (MFI)</td>
<td>Calculate magnetic field strength due to various shapes</td>
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<td>MFI due to circular, square and solenoid current</td>
<td>Calculate MFI due to circular, square and solenoid current</td>
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<td>magnetic flux, magnetic flux density and MFI</td>
<td>Apply the carrying wire Relation between magnetic flux, magnetic flux density</td>
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<td>Maxwell’s second Equation</td>
<td>Derive Maxwell’s second Equation physical significance and to apply it</td>
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<td>Ampere’s circuital law</td>
<td>Derive ampere’s law, Its applications.</td>
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<td>Point form of Ampere’s circuital law</td>
<td>Solve point form of Ampere’s circuital law and to Analyze the point form of Ampere’s circuital law</td>
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<td>34</td>
<td>Maxwell’s third equation</td>
<td>Explain Maxwell's third equation physical significance</td>
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</table>

**UNIT - IV**

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<tr>
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<td>Magneto Statics</td>
<td>Explain force in Magnetic fields and Magnetic Potential and to Observe the Magnetic force</td>
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<tr>
<td>36</td>
<td>Lorentz force equation</td>
<td>Derive the Lorentz force equation and to Apply the moving charges in a magnetic field</td>
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<td>Force on a current element in a</td>
<td>Calculate the force on a current element in a magnetic field</td>
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<td>38</td>
<td>Force on a straight and a long current carrying conductor in a magnetic field</td>
<td>Derive force on a straight and a long current carrying conductor in a magnetic field</td>
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<tr>
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<td>Problems on Lorentz force equation</td>
<td>Apply the Lorentz force equation to moving charges in a magnetic field</td>
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<td>Concentrate Force between two straight long and parallel current carrying conductors</td>
<td>Derive the force between two straight long and parallel current carrying conductors to understand the force between two straight long and parallel current carrying conductors</td>
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<td>41</td>
<td>Problems on magnetic fields</td>
<td>Apply the bio savers law equation to moving charges in a magnetic field</td>
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<td>42</td>
<td>Magnetic dipole and dipole moment</td>
<td>Apply the Magnetic dipole and dipole moment</td>
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<td>43</td>
<td>Differential current loop as a magnetic dipole</td>
<td>Explain the differential current loop as a magnetic dipole</td>
</tr>
<tr>
<td>44</td>
<td>Torque on a current loop placed in a magnetic field</td>
<td>Solve the Torque on a current loop in magnetic field Scalar Magnetic</td>
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<td>45</td>
<td>Vector magnetic potential</td>
<td>Apply the vector magnetic potential and its properties</td>
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<tr>
<td>46</td>
<td>Vector Poisson’s equations</td>
<td>Apply the vector Poisson’s equations to single and two dimensional</td>
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<tr>
<td>47</td>
<td>Self and Mutual inductance</td>
<td>Derive the self and Mutual inductance for various cases</td>
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<tr>
<td>48</td>
<td>Neumann’s formulae</td>
<td>Develop the Neumann’s formula and to Solve Neumann’s formula</td>
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<td>49</td>
<td>Problems on Neumann’s formulae</td>
<td>Solve Neumann’s formula Problems on Neumann’s formula</td>
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<td>Determination of self-inductance of a solenoid and torrid</td>
<td>Derive the determination of self-inductance of a solenoid and torrid</td>
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<tr>
<td>51</td>
<td>Mutual inductance between a long wire</td>
<td>Derive mutual inductance between a straight long wire</td>
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<td>52</td>
<td>Introduction to permanent magnets, their characteristics</td>
<td>Derive energy stored and density in a magnetic field</td>
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<td>53</td>
<td>Magnetic force exerted in motors</td>
<td>Explain magnetic field laws right hand rule</td>
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<td>54</td>
<td>Magnetic force exerted in generators</td>
<td>Explain magnetic field laws left hand rule</td>
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**UNIT – V**

| 53 | time varying Fields: Time varying fields | Explain the Time varying fields |
| 54 | laws of electromagnetic induction | Develop the electromagnetic induction |
| 55 | Maxwell’s fourth equation, Curl (E)=−B/t | Derive Maxwell’s fourth equation |
| 56 | Maxwell’s equations for time varying | Apply Maxwell's equations for time varying |
| 57 | Displacement current | Explain Displacement current |
| 58 | Modified Maxwell’s equations | Apply Maxwell’s equations |
| 59 | Pointing theorem | Derive Pointing theorem |
| 60 | Application of pointing theorem | Apply Pointing theorem |
| 61 | Electro static and magnetic filed applications | Explain Electro static and magnetic filed applications |

**4. COURSE PREREQUISITES**

1. Mathematics II
2. Physics II
5) CO’S, PO’S MAPPING:

CO&PO Mappings

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## 6. COURSE INFORMATION SHEET (CIS)

(a) Course description

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<td>COURSE: Electromagnetic fields</td>
<td>YEAR: II SEM: I CREDITS: 4</td>
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<td>COURSE AREA/DOMAIN: Electrical</td>
<td>CONTACT HOURS: 4+1 (L+T) hours/Week.</td>
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(b) Syllabus

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<tr>
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<th>CLASSES</th>
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<tbody>
<tr>
<td>I</td>
<td><strong>Electrostatics:</strong> Electrostatic Fields, Coulomb’s Law, Electric Field Intensity (EFI) – EFI due to a line and a surface charge, Work done in moving a point charge in an electrostatic field, Electric Potential, Properties of potential function, Potential gradient, Guass’s law, Application of Guass’s Law, Maxwell’s first law, div ( D )=v – Laplace’s and Poisson’s equations, Solution of Laplace’s equation in one variable, Electric dipole, Dipole moment, potential and EFI due to an electric dipole, Torque on an Electric dipole in an electric field</td>
<td>16</td>
</tr>
<tr>
<td>II</td>
<td><strong>Dielectrics &amp; Capacitance:</strong> Behavior of conductors in an electric field, Conductors and Insulators, Electric field inside a dielectric material, polarization, Conductor and Dielectric, Dielectric boundary conditions, Capacitance, Capacitance of parallel plots, spherical co-axial capacitors with composite dielectrics, Energy stored and energy density in static field, dielectrics and their properties</td>
<td></td>
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<tr>
<td>III</td>
<td><strong>Magneto statics:</strong> Static magnetic fields - biot-savarts law, magnet f MFI due to a straight current carrying filament, MFI due circular square and solenoid current carrying wire, relation between magnetic flux density and MFI Maxwell’s second equation, div(B)=0. Ampere’s circuital law and its applications viz. MFI due to an infinite sheet of current and a long current carrying filament – Point form of Ampere’s circuital law, Maxwell’s third equation, Curl (H)=Jc</td>
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<tr>
<td>IV</td>
<td><strong>Force in Magnetic fields and Magnetic Potential:</strong> Magnetic force, Moving charges in a Magnetic field, Lorentz force equation, force on a current element in a magnetic field, Force on a straight and a long current carrying conductor in a magnetic field, Force between Two straight long and parallel current carrying conductors, Magnetic dipole and dipole moment, a differential current loop as a magnetic dipole, Torque on a current loop placed in a magnetic field Scalar Magnetic potential and its limitations – vector magnetic potential and its properties, vector magnetic potential due to simple configurations, vector Poisson’s equations. Self and Mutual inductance, Neumann’s formulae, determination of self-inductance of a solenoid and toroid and mutual inductance between a straight long wire and a square loop wire in the same plane, energy stored and density in a magnetic field. Introduction to permanent magnets, their characteristics and applications</td>
<td></td>
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</table>
| V | **Time Varying Fields:** Time varying fields, Faraday’s laws of electromagnetic induction, Its integral and point forms, Maxwell’s fourth equation, Curl (E)=-B/t, Statically and }
### Dynamically induced EMFs, Simple problems, Modification of Maxwell’s equations for time varying fields, Displacement current

| Contact classes for syllabus coverage | 61 |
| Lectures beyond syllabus | 01 |
| Classes for gaps & Add-on classes | 01 |
| Total No. of classes | 63 |

#### (c) Gaps in syllabus

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<td>Magnetic force exerted in motors and generators</td>
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#### (d) Topics beyond Syllabus

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<td>Time varying fields applications</td>
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#### (e) Web Source References

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<tbody>
<tr>
<td>1</td>
<td>nptel.ac.in/courses/112106121/1</td>
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<td>2</td>
<td>ebooks.library.cornell.edu/k/emfddl/pdf/016_002.pdf</td>
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<tr>
<td>3</td>
<td><a href="http://nptel.ac.in/courses/108108076/22">http://nptel.ac.in/courses/108108076/22</a></td>
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</tbody>
</table>
(f) Delivery / Instructional Methodologies:

| ☑ CHALK & TALK | ☑ STUD. ASSIGNMENT | ☑ WEB RESOURCES |
| ☑ LCD/SMART BOARDS | ☑ STUD. SEMINARS | ☐ ADD-ON COURSES |

(g) Assessment Methodologies - Direct

| ☑ Assignments | ☑ Stud. Seminars | ☑ Tests/Model Exams | ☑ Univ. Examination |
| ☐ Add-On Courses | ☐ Others | | |

(h) Assessment Methodologies - Indirect

| ☑ Assessment Of Course Outcomes (By Feedback, Once) | ☑ Student Feedback On Faculty (Twice) |
| ☐ Assessment Of Mini/Major Projects By Others | ☐ Others |
(i) Text books and References

**Text Books**

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Title</th>
<th>Author(s)</th>
<th>Publisher</th>
<th>Edition</th>
<th>Year</th>
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**Suggested / Reference Books**

<table>
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<tr>
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<th>Author(s)</th>
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7. MICRO LESSON PLAN

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<th>Schedule data</th>
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<td>3.</td>
<td>Coulomb’s Law – Electric Field Intensity</td>
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<td>(EFI)</td>
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<tr>
<td>4</td>
<td>EFI due to a line and a surface charge</td>
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<tr>
<td>5</td>
<td>Work done in moving a point charge in an electrostatic field</td>
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<td>6</td>
<td>Electric Potential</td>
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<td>7</td>
<td>Properties of potential function – Potential gradient</td>
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<td>8</td>
<td>Gauss’s law – Application of Gauss’s Law</td>
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<td>Maxwell’s first law, Laplace’s and Poison’s equations</td>
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<td>Solution of Laplace’s equation in one variable</td>
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<td>Behavior of conductors in an electric field</td>
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<td>Conductors and Insulators</td>
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<td>30 Biot-Savart’s law</td>
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<td>41 Moving charges in a Magnetic field – Lorentz force equation</td>
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<td>42 Force on a current element in a magnetic field</td>
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<td>43 Force on a conductor in a magnetic field</td>
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<td>44 Force between two straight long and parallel current carrying conductors</td>
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<td>45 Magnetic dipole and dipole moment</td>
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<td>46 Current loop as a magnetic dipole</td>
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<td>48 Vector magnetic potential and its properties</td>
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<td>56</td>
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<td>57</td>
<td>Maxwell’s fourth equation, Statically and Dynamically induced EMFs</td>
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<td>58</td>
<td>Maxwell’s equations for time varying fields</td>
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<td>59</td>
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<td>Application of pointing theorem</td>
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<td>61</td>
<td>Electro static and magnetic filed applications</td>
</tr>
<tr>
<td>62</td>
<td>Pointing theorem</td>
</tr>
<tr>
<td>63</td>
<td>Applications of time varying fields</td>
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### 8) Teaching Schedule

<table>
<thead>
<tr>
<th>Subject</th>
<th>ELECTROMAGNETIC FIELDS</th>
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<tbody>
<tr>
<td>Text Books</td>
<td>(to be purchased by the Students)</td>
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<tr>
<td>Reference</td>
<td>Books</td>
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<tr>
<td>I</td>
<td>Electrostatics</td>
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<td>Guass’s Law – Maxwell’s first law</td>
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<td>Electric dipole – Dipole moment</td>
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<td>II</td>
<td>Dielectrics &amp; Capacitance</td>
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<tr>
<td>III</td>
<td>Magneto Statics</td>
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<td>Ampere’s Law &amp; Application</td>
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<tr>
<td>IV</td>
<td>Force in Magnetic fields and Magnetic Potential</td>
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<td>Self and Mutual inductance</td>
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<tr>
<td>V</td>
<td>Time Varying Fields: Time varying fields</td>
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<tr>
<td>Contact classes for syllabus coverage</td>
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<tr>
<td>Lecture beyond syllabus and gaps (PPT &amp; NPTEL)</td>
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<td>Total No. of classes</td>
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9. Unit-wise Hand written notes (Soft copy and Hard copy)
10. OHD/LCD SHEETS /CDS/DVDS/PPT (SOFT/HARD COPIES)
11. University Previous Question papers

Code No: 113BY
JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, May/June - 2015
ELECTROMAGNETIC FIELDS
(Electrical and Electronics Engineering)

Time: 3 Hours Max. Marks: 75

Note: This question paper contains two parts A and B. Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks.

PART- A
(25 Marks)

1.a) Write any two relations between E and V. [2M]

b) Plane z=10 m carries charge 20 nC/m². Find the electric field intensity at the origin. [3M]

c) State Continuity Equation. [2M]

d) Define conduction and convection current densities. [3M]

e) Justify div (B) =0. [2M]

f) Define magnetic field intensity, magnetic flux density and express the relation between them. [3M]

g) Define scalar magnetic potential and vector magnetic potential. [2M]

h) Mention the characteristics of permanent magnets. [3M]

i) State Faraday’s laws of electromagnetic induction. [2M]

j) Express four Maxwell’s equations in point form. [3M]

PART-B
(50 Marks)

2.a) Derive the expression for work done in moving a point charge in an electrostatic field. [6+4]

b) Let \( D = 2y^2z^2 + 3xy^2z^2 + 2xyz \) pC/m² in free space. Find the total electric flux passing through the surface \( x=2, 0 \leq y \leq 2, 0 \leq z \leq 2 \) in a direction away from the origin.

OR

3.a) Obtain the electric field intensity \( E \) at a point when an infinite line charge is placed along z-axis by applying Gauss’s Law. [5+5]

b) Find the flux density \( \mathbf{D} \) at appoint A(6,4,-5) caused by a point charge of 20 mC at the origin.

4. Derive the expression for energy stored and energy density in a static electric field. [10]

OR
5. a) Derive the equation of continuity.
   b) Given the vector current density \( J = 10\rho^2 z a_\rho - 4\rho cos^2 \phi a_\phi \) A/m², determine the total current flowing outward through the circular band \( \rho = 3, 0 < \phi < 2\pi, 2 < z < 2.8 \).

6. Evaluate the closed line integral of \( H \) about the rectangular path P1(2,3,4) to P2(4,3,4) to P3(4,3,1) to P4(2,3,1) to P1, given \( H = 3z a_x - 2x^3 a_z \) A/m.

   OR

7. State Biot‐Savart’s law and obtain the expression for magnetic field intensity due to an infinitely long straight filament carrying current by applying Biot‐Savart’s Law.

8. If \( A = 10\rho^{1.5} a_\rho \) Wb/m in free space find (a) Magnetic field intensity \( H \).
   (b) Current density \( J \) and current \( I \) (c), show that \( \oint H \cdot dl = I \) for circular path with \( \rho = 1 \).

   OR

9. Obtain the expression for magnetic torque in terms of magnetic dipole moment.

10. In free space \( E = 20 \cos(\omega t - 50x)a_x \) V/m. Calculate (a) Displacement current density and (b) Magnetic field intensity \( H \).

   OR

11. a) Find the conduction and displacement current densities in a material having conductivity of \( 10^{-5} \) S/m and \( \varepsilon_r = 2.5 \) if the electric field in the material is \( E = 5.0 \times 10^{-6} \sin(9.0 \times 10^6 t) V/m \).
   b) Explain the significance of Displacement current.
<table>
<thead>
<tr>
<th>Q.NO</th>
<th>QUESTION</th>
<th>CO mapping</th>
<th>Blooms taxonomy level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Apply gauss law and find E.F.I due to Infinite long wire?</td>
<td>CO2</td>
<td>Apply</td>
</tr>
<tr>
<td>2</td>
<td>Derive expression for torque due to dipole?</td>
<td>CO2</td>
<td>Analyze</td>
</tr>
<tr>
<td>3</td>
<td>Describe the dielectric boundary condition?</td>
<td>CO1</td>
<td>understand</td>
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<tr>
<td>4</td>
<td>Derive expression for M.F.I due to solenoid</td>
<td>CO1</td>
<td>understand</td>
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## II Mid Examinations

**Marks:** 10

**Year-Sem & Branch:** II-I

**Duration:** 60 Min

**Subject:** ET

**Date & Session:**

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<thead>
<tr>
<th>Q.NO</th>
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<th>CO mapping</th>
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<tr>
<td>1</td>
<td>Explain modified Maxwell’s equations</td>
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<td>Understand</td>
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<tr>
<td>2</td>
<td>Explain Faraday’s laws of electromagnetic induction</td>
<td>CO1</td>
<td>Understand</td>
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<tr>
<td>3</td>
<td>Derive Neman’s formula for self and mutual inductance</td>
<td>CO2</td>
<td>Analyze</td>
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<tr>
<td>4</td>
<td>Explain working principle of generator?</td>
<td>CO1</td>
<td>Understand</td>
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</table>

Answer **ANY TWO** of the following questions: $2 \times 5 = 10$

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13. MID exam Objective Question papers
Choose the correct alternative:

1. Force on a stationary or moving electric charge Q in electric field E is given by
   (A) \(E/Q^2\)        (B) \(EQ\)        (C) \(E/Q\)        (D) \(E^2/Q\)

2. The electric flux lines from a charged conducting body such that the angle between the surface of the conductor and the flux line is
   (A) \(\pi/2\)        (B) \(\pi/3\)        (C) \(\pi/6\)        (D) \(\pi/4\)

3. The electric potential has
   (A) Only magnitude  (B) only direction  (C) neither magnitude nor direction  (D) none of the above

4. Potential at a point distance ‘r’ from the dipole varies as
   (A) \(1/r\)        (B) \(1/r^2\)        (C) \(1/r^3\)        (D) independent of \(r\)

5. For perfect conductors the charge density inside the surface is
   (A) maximum  (B) zero  (C) one  (D) minimum

6. Capacitance of two concentric conducting spheres of radius ‘a’ and ‘b’ is given by
   (A) \(4\pi\varepsilon_o b\)  (B) \(4\pi\varepsilon_o ab/(b-a)\)  (C) \(4\pi\varepsilon_o a\)  (D) \(4\pi\varepsilon_o a\)

7. Boundary condition for free space and dielectric is
   (A) \(D_1 - D_2 = \rho_s\)  (B) \(D_{\text{in}} = \rho_s\)  (C) \(E_{\text{out}} = E_{\text{in}}\)  (D) none of these

8. A void is formed in a parallel plate capacitor which is filled with a dielectric of \(\varepsilon_r = 4\). If the electric stress in the dielectric is 20 KV/cm, then the electric stress in the void is
   (A) 20 KV/cm  (B) 80 KV/cm  (C) 5 KV/cm  (D) 200 KV/cm

9. If a conductor is lying on the z-axis, to find the magnetic field in cylindrical coordinates around the conductor, the magnetic field should be along
   (A) \(a_z\)  (B) \(a_r\)  (C) \(a_\theta\)  (D) \(a_z\)

10. The field intensity due to infinite line current is inversely proportional to
    (A) Flux  (B) distance  (C) charge  (D) square of distance

Cont....2
II Fill in the Blanks

11. Magnetic field intensity in terms of magnetic flux density is given by _________________.
12. The equation that indicates that the magnetic flux lines are continuous is _________________.
13. Ohm’s law in point form is _________________.
14. Boundary conditions for a perfect dielectric material is derived from ________________ law.
15. The poisson’s equation for the potential V is _________________.
16. For a given dipole, if the angle between \( \mathbf{p} \) and \( \mathbf{E} \) is zero, then the torque on the dipole is _________________.
17. In an electric field, the maximum value of rate of change of potential with respect to the distance is called _________________.
18. Electric flux originates at ________________ and terminates at _________________.
19. Gauss’s law in integral form is _________________.
20. The unit of dipole moment is _________________.

-00-
14. Assignment topics with materials

UNIT-1
Electromagnetic fields

1. State and explain the couloms law?

Coulomb’s Law states that the force between two point charges $Q_1$ and $Q_2$ is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

Point charge is a hypothetical charge located at a single point in space. It is an idealized model of a particle having an electric charge.

Mathematically, \[ F = \frac{kQ_1Q_2}{R^2}, \]

where $k$ is the proportionality constant.

In SI units, $Q_1$ and $Q_2$ are expressed in Coulombs (C) and $R$ is in meters. Force $F$ is in Newton’s (N) and, $\varepsilon_0$ is called the permittivity of free space.

\[ k = \frac{1}{4\pi\varepsilon_0} \]

(We are assuming the charges are in free space. If the charges are any other dielectric medium, we will use $\varepsilon = \varepsilon_0\varepsilon_r$ instead where $\varepsilon_r$ is called the relative permittivity or the dielectric constant of the medium).

Therefore

\[ F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1Q_2}{R^2} \]

As shown in the Figure let the position vectors of the point charges $Q_1$ and $Q_2$ are given by $\vec{r}_1$ and $\vec{r}_2$. Let $\vec{F}_{12}$ represent the force on $Q_1$ due to charge $Q_2$.

\[ \vec{F}_{12} \]

Coulomb's Law

The charges are separated by a distance of $R = |\vec{r}_1 - \vec{r}_2| = |\vec{r}_2 - \vec{r}_1|$. We define the unit vectors as
\[ \hat{d}_{12} = \frac{\hat{r}_2 - \hat{r}_1}{R} \quad \text{and} \quad \hat{d}_{21} = \frac{\hat{r}_1 - \hat{r}_2}{R} \]

\[ \vec{F}_{12} = \frac{Q_1 Q_2}{4 \pi \varepsilon_0 R^2} \hat{d}_{12} = \frac{Q_1 Q_2}{4 \pi \varepsilon_0 R^2} \frac{\hat{r}_2 - \hat{r}_1}{|\hat{r}_2 - \hat{r}_1|^3} . \]

Similarly the force on \( Q_1 \) due to charge \( Q_2 \) can be calculated and if \( \vec{F}_{21} \) represents this force then we can write \( \vec{F}_{21} = -\vec{F}_{12} \).

When we have a number of point charges, to determine the force on a particular charge due to all other charges, we apply principle of superposition. If we have \( N \) number of charges \( Q_1, Q_2, \ldots, Q_N \) located respectively at the points represented by the position vectors \( \hat{r}_1, \hat{r}_2, \ldots, \hat{r}_N \), the force experienced by a charge \( Q \) located at \( \hat{r} \) is given by,

\[ \vec{F} = \frac{Q}{4 \pi \varepsilon_0} \sum_{i=1}^{N} Q_i \frac{(\hat{r} - \hat{r}_i)}{|\hat{r} - \hat{r}_i|^3} \]

**Electric Field**

The electric field intensity or the electric field strength at a point is defined as the force per unit charge. That is

\[ \vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q} \]

### 2. Write short notes on Electric field intensity?

The electric field intensity \( \vec{E} \) at a point \( \hat{r} \) (observation point) due a point charge \( Q \) located at \( \hat{r} \) (source point) is given by:

\[ \vec{E} = \frac{Q (\hat{r} - \hat{r}')}{|\hat{r} - \hat{r}'|^3} \]

For a collection of \( N \) point charges \( Q_1, Q_2, \ldots, Q_N \) located at \( \hat{r}_1, \hat{r}_2, \ldots, \hat{r}_N \), the electric field intensity at point \( \hat{r} \) is obtained as

\[ \vec{E} = \frac{1}{4 \pi \varepsilon_0} \sum_{i=1}^{N} Q_i \frac{(\hat{r} - \hat{r}_i)}{|\hat{r} - \hat{r}_i|^3} \]

The expression can be modified suitably to compute the electric filed due to a continuous distribution of charges.

In figure 2.2 we consider a continuous volume distribution of charge \( \square(t) \) in the region denoted as the source region.

For an elementary charge \( dQ = \rho(\hat{r}') dV' \), i.e. considering this charge as point charge, we can write the field expression as:
When this expression is integrated over the source region, we get the electric field at the point $P$ due to this distribution of charges. Thus the expression for the electric field at $P$ can be written as:

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r') \vec{r} - \vec{r'}}{r^3} dV'$$

Similar technique can be adopted when the charge distribution is in the form of a line charge density or a surface charge density.

$$\vec{E}(r) = \frac{1}{2\pi\varepsilon_0} \int \frac{\rho_x(r') \vec{r} - \vec{r'}}{r^3} dl'$$

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_s(r') \vec{r} - \vec{r'}}{r^3} ds'$$

Coulomb's Law states that the force between two point charges $Q_1$ and $Q_2$ is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. Mathematically, $F = \frac{kQ_1Q_2}{R^2}$, where $k$ is the proportionality constant.

3. State and explain the gauss law?

Gauss's Law: Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.
Let us consider a point charge $Q$ located in an isotropic homogeneous medium of dielectric constant $\varepsilon$. The flux density at a distance $r$ on a surface enclosing the charge is given by

$$\overrightarrow{D} = \varepsilon \overrightarrow{E} = \frac{Q}{4\pi r^2} \hat{a}_r$$

If we consider an elementary area $dA$, the amount of flux passing through the elementary area is given by

$$d\psi = \overrightarrow{D}.dA = \frac{Q}{4\pi r^2} dscos\theta$$

$$dscos\theta = d\Omega$$

But $\frac{r^2}{d\Omega}$ is the elementary solid angle subtended by the area $dA$ at the location of $Q$. Therefore we can write

$$d\psi = \frac{Q}{4\pi} d\Omega$$

Moreover

$$\psi = \oint d\psi = \frac{Q}{4\pi} \int d\Omega = Q$$

4. **Explain the applications of gauss law?**

Gauss's law is particularly useful in computing $\overrightarrow{E}$ or $\overrightarrow{D}$ where the charge distribution has some symmetry. We shall illustrate the application of Gauss's Law with some examples. An Infinite line charge

As the first example of illustration of use of Gauss's Law, let consider the problem of determination of the electric field produced by an infinite line charge of density $\lambda$ C/m. Let us consider a line charge positioned along the z-axis as shown in Fig. 2.4(a) (next slide). Since the line charge is assumed to be infinitely long, the electric field will be of the form as shown in Fig. b.

If we consider a close cylindrical surface as shown in Fig. 2.4(a), using Gauss's theorem we can write,
\[ \rho_{2l} = Q = \oint \vec{E} \cdot d\vec{s} = \oint \varepsilon_0 \vec{E} \cdot d\vec{s} + \oint \varepsilon_0 \vec{E} \cdot d\vec{s} + \oint \varepsilon_0 \vec{E} \cdot d\vec{s} \]

Considering the fact that the unit normal vector to areas \( S_1 \) and \( S_3 \) are perpendicular to the electric field, the surface integrals for the top and bottom surfaces evaluates to zero. Hence we can write, 
\[ \rho_{2l} = \varepsilon_0 E \cdot 2\pi \alpha \]

Electric fields

Fig : Infinite
Line Charge

\[ \vec{E} = \frac{\rho_L}{2\pi \varepsilon_0 \rho} \hat{\rho} \]

Infinite Sheet of Charge

As a second example of application of Gauss’s theorem, we consider an infinite charged sheet covering the \( x-z \) plane as shown in figure 2.5.

Assuming a surface charge density \( \rho_s \) for the infinite surface charge, if we consider a cylindrical volume having sides \( \Delta s \) placed symmetrically as shown in figure 5, we can write:

\[ \text{Fig : 2.5 Infinite Sheet of Charge} \]

It may be noted that the electric field strength is independent of distance. This is true for the infinite plane of charge; electric lines of force on either side of the charge will be perpendicular to the sheet and extend to infinity as parallel lines. As number of lines of force per unit area gives the strength of the field, the field becomes independent of distance. For a finite charge sheet, the field will be a function of distance.

3. Uniformly Charged Sphere

Let us consider a sphere of radius \( r_0 \) having a uniform volume charge density of \( \sigma \) \( \text{C/m}^3 \). To determine \( \vec{D} \) everywhere, inside and outside the sphere, we construct Gaussian surfaces of radius \( r < r_0 \) and \( r > r_0 \) as shown in Fig. (a) and Fig (b).

\[ \text{Fig : Uniformly Charged Sphere} \]

For the region \( r \leq r_0 \); the total
enclosed charge will be

\[ Q_{\text{en}} = \rho_v \frac{4}{3} \pi r^3 \]

By applying Gauss's theorem,

\[ \oint \vec{D} \cdot d\vec{s} = \int \int \int_{\Omega} \vec{D}_{r} r^2 \sin \theta d\theta d\phi = 4 \pi r^2 D_r = Q_{\text{en}} \]

Therefore

\[ \vec{D} = r \frac{\rho_v \hat{\epsilon}_r}{3}, \quad 0 \leq r \leq r_0. \]

For the region \( r \geq r_0 \); the total enclosed charge will be

\[ Q_{\text{en}} = \rho_v \frac{4}{3} \pi r_0^3 \]

By applying Gauss's theorem,

\[ \vec{D} = \frac{r_0^3}{2 \pi^2} \rho_v \hat{\epsilon}_r, \quad r \geq r_0 \]

5. **Derive the Maxwell’s first law, \( \text{div} (\vec{D}) = \nabla \cdot \vec{E} \) equations?**

Equations give the relationship among the field quantities in the static field. For time varying case, the relationship among the field vectors written as

\[ \nabla \times \vec{B} = -\frac{\partial \vec{E}}{\partial t} \]

\[ \nabla \times \vec{H} = \vec{J} \]

\[ \nabla \cdot \vec{D} = \rho \]

\[ \nabla \cdot \vec{B} = 0 \]

In addition, from the principle of conservation of charges we get the equation of
continuity

\[ \nabla \mathbf{J} = -\frac{\partial \rho}{\partial t} \]

The equation must be consistent with equation

The divergence of the vector flu density \( A \) is the outflow of flu from a small closed surface per unit volume as the volume shrinks to zero. A positive divergence for any vector quantity indicates a source of that vector quantity at that point. Similarly, a negative divergence indicates a sink. Because the divergence of the water velocity above is zero, no source or sink exists. The expanding air, however, produces a positive divergence of the velocity, and each interior point may be considered a source.

Writing with our new term, we have

\[
\text{div } D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \quad \text{(rectangular)}
\]

This expression is again of a form that does not involve the charge density. It is the result of applying the definition of divergence to a differential volume element in rectangular coordinates.

If a differential volume unit \( \rho \, dr \, d\varphi \, dz \) in cylindrical coordinates, or \( r \, \sin \theta \, dr \, d\theta \, dz \)

d\( \theta \, d\varphi \) in spherical coordinates, had been chosen, expressions for divergence involving the components of the vector in the particular coordinate system and involving partial derivatives with respect to the variables of that system would have been obtained. These expressions are obtained in Appendix A and are given here for convenience:

\[
\text{div } D = \frac{1}{\rho} \frac{d}{dr} (\rho D_r) + \frac{1}{\rho} \frac{\partial D_\varphi}{\partial \varphi} + \frac{\partial D_z}{\partial z} \quad \text{(cylindrical)}
\]

\[
\text{div } D = \frac{1}{r^2} \frac{d}{dr} (r^2 D_r) + \frac{1}{r \sin \theta} \frac{d}{d\theta} (\sin \theta D_\theta) + \frac{1}{r \sin \theta} \frac{\partial D_\varphi}{\partial \varphi} \quad \text{(spherical)}
\]

These relationships are also shown inside the back cover for easy reference. It should be noted that the divergence is an operation which is performed on a vector, but that the result is a scalar. We should recall that, in a somewhat similar way, the dot or scalar
product was a multiplication of two vectors which yielded a scalar.

6. Define the electric dipole & Explain EFI due to an electric dipole?

Electric Dipole

An electric dipole consists of two point charges of equal magnitude but of opposite sign and separated by a small distance.

As shown in figure 2.11, the dipole is formed by the two point charges $Q$ and $Q$ separated by a distance $d$, the charges being placed symmetrically about the origin.

Let us consider a point $P$ at a distance $r$, where we are interested to find the field.

The potential at $P$ due to the dipole can be written as:

$$ V = \frac{1}{4 \pi \varepsilon_0} \left[ \frac{Q}{r_1} - \frac{Q}{r_2} \right] = \frac{Q}{4 \pi \varepsilon_0} \left[ \frac{r_2 - r_1}{r_1 r_2} \right] $$

When $r_1$ and $r_2 >> d$, we can write

$$ r_2 - r_1 = 2 \times \frac{d}{2} \cos \theta = d \cos \theta $$

and $r_1 \approx r_2 \approx r$.

Therefore,

$$ V = \frac{Q}{4 \pi \varepsilon_0} \frac{d \cos \theta}{r^2} $$

We can write,
\[ \vec{P} = Qd \]

The quantity \( \vec{P} = Qd \) is called the dipole moment of the electric dipole.

Hence the expression for the electric potential can now be written as:

\[ V = \frac{\vec{P} \cdot \hat{r}}{4\pi \varepsilon_0 r^2} \]

It may be noted that while potential of an isolated charge varies with distance as \( 1/r \) that of an electric dipole varies as \( 1/r^2 \) with distance.

If the dipole is not centered at the origin, but the dipole center lies at \( \vec{r} \), the expression for the potential can be written as:

\[ V = \frac{\vec{P} \cdot (\vec{r} - \vec{r}')}{{\varepsilon_0}^2 |\vec{r} - \vec{r}'|^3} \]

The electric field for the dipole centered at the origin can be computed as

\[ \vec{E} = -\nabla V = - \left[ \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} \right] = \frac{Qd \cos \theta}{2\pi \varepsilon_0 r^3} \hat{r} + \frac{Qd \sin \theta}{4\pi \varepsilon_0 r^3} \hat{\theta} = \frac{Qd}{4\pi \varepsilon_0 r^3} \left( 2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right) \]

\[ \vec{E} = \frac{\vec{P}}{4\pi \varepsilon_0 r^3} \left( 2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right) \]

7. **Explain the dipole moment in electrostatic fields?**

The electric dipole moment is a measure of the separation of positive and negative electrical charges within a system, that is, a measure of the system's overall polarity. The electric field strength of the dipole is proportional to the magnitude of dipole moment. The SI units for electric dipole moment are coulomb-meter (C-m), however the most commonly used unit is the Debye (D).
Theoretically, an electric dipole is defined by the first-order term of the multiple expansions, and consists of two equal and opposite charges infinitely close together. This is unrealistic, as real dipoles have separated charge. However, because the charge separation is very small compared to everyday lengths, the error introduced by treating real dipoles like they are theoretically perfect is usually negligible. The direction of dipole is usually defined from the negative charge towards the positive charge.

\[ P = qd \]

An object with an electric dipole moment is subject to a torque \( \tau \) when placed in an external electric field. The torque tends to align the dipole with the field. A dipole aligned parallel to an electric field has lower potential energy than a dipole making some angle with it. For a spatially uniform electric field \( E \), the torque is given by:

\[ T = P \times E \]

Where \( p \) is the dipole moment, and the symbol "\( \times \)" refers to the vector cross product. The field vector and the dipole vector define a plane, and the torque is directed normal to that plane with the direction given by the right-hand rule.

A dipole orients co- or anti-parallel to the direction in which a non-uniform electric field is increasing (gradient of the field) will not experience a torque, only a force in the direction of its dipole moment. It can be shown that this force will always be parallel to the dipole moment regardless of co- or anti-parallel orientation of the dipole.

8. **Derive the potential gradient** \( \nabla \phi = 0 \)?

The potential of a single point charge at the origin depends solely on the radial distance to the observation point \( A \).
the potential difference \( V_{AB} \) between points A and B depends solely on their radial
distances from the origin.

\[
V_{AB} = \int_{A}^{B} \mathbf{E} \cdot d\mathbf{L} = \int_{A}^{B} \frac{Q}{4\pi\varepsilon r^2} \mathbf{a}_r \cdot d\mathbf{r} = \frac{Q}{4\pi\varepsilon} \left( \frac{1}{r_A} - \frac{1}{r_B} \right)
\]

angular positions, \( \theta \) and \( \phi \), of observation points do not matter. path of integration does
not matter – integrand has only \( r \) component and \( r \) dependence. if path of integration is
closed – potential difference is zero.

\[
V_{AA} = \oint_{c} \mathbf{E} \cdot d\mathbf{L} = \frac{Q}{4\pi\varepsilon} \left( \frac{1}{r_A} - \frac{1}{r_A} \right) = 0
\]

Conservative property of potential follows from superposition and conservative property
of potential of point charge. If work along a closed path is zero for a single point
charge, it will be zero for any collection of charges. electrostatic potential taken on a
closed integration path is zero.

- remember the del vector operator from

\[
\nabla = \frac{\partial}{\partial x} \mathbf{a}_x + \frac{\partial}{\partial y} \mathbf{a}_y + \frac{\partial}{\partial z} \mathbf{a}_z
\]

\[
\Rightarrow \mathbf{E} = -\nabla V, \text{ V/m} \quad \nabla V = \text{grad} V
\]

9. Explain the behavior of Conductors in Electric Field?

In a conductor, electric current can flow freely, in an insulator it cannot. Metals such as
copper typify conductors, while most non-metallic solids are said to be good insulators,
having extremely high resistance to the flow of charge through them. "Conductor" implies that the outer electrons of the atoms are loosely bound and free to move through the material. Most atoms hold on to their electrons tightly and are insulators. In copper, the valence electrons are essentially free and strongly repel each other. Any external influence which moves one of them will cause a repulsion of other electrons which propagates, "domino fashion" through the conductor.

Simply stated, most metals are good electrical conductors, most nonmetals are not. Metals are also generally good heat conductors while nonmetals are not.

Conductors have electrons which are not bound tightly in their atoms. These are free to move within the conductor. However, there is no net transfer of electrons (charges) from one part of the conductor to the other in the absence of any applied electric field. The conductor is said to be in electrostatic equilibrium.

When a conductor placed in an external electric field $E$, the free electrons are accelerated in a direction opposite to that of the electric field. This results in buildup of electrons on the surface ABCD of the conductor.

The surface FGHK becomes positively charged because of removal of electrons. These charges (-ve on surface ABCD and +ve on surface FGHK) create their own fields, which are in a direction opposite to $E$.

![Diagram of electrostatic shielding](image)

It is a phenomenon of protecting a certain region of space from external electric fields. To protect delicate instruments from external electric fields, they are enclosed in hollow conductors. That is why in a thunder storm accompanied by lightning, it is safer to be inside a car or a bus than outside. The metallic body of the car or bus provides electrostatic shielding from lightning.
UNIT-II
DIELECTRIC AND CAPACITANCE

1. Explain the phenomenon of parallel plate capacitors?
   Capacitance and Capacitors
   We have already stated that a conductor in an electrostatic field is an Equi-potential body and any charge given to such conductor will distribute themselves in such a manner that electric field inside the conductor vanishes. If an additional amount of charge is supplied to an isolated conductor at a given potential, this additional charge will increase the surface charge density $\rho_s$. Since the potential of the conductor is given by $V = \frac{1}{4\pi \varepsilon_0} \int \frac{\rho_s ds}{r}$, the potential of the conductor will also increase maintaining the ratio $\frac{Q}{V}$ same. Thus we can write $C = \frac{Q}{V}$ where the constant of proportionality $C$ is called the capacitance of the isolated conductor. SI unit of capacitance is Coulomb/ Volt also called Farad denoted by F. It can be seen that if $V=1$, $C = Q$. Thus capacity of an isolated conductor can also be defined as the amount of charge in Coulomb required to raise the potential of the conductor by 1 Volt.
   Of considerable interest in practice is a capacitor that consists of two (or more) conductors carrying equal and opposite charges and separated by some dielectric media or free space. The conductors may have arbitrary shapes. A two-conductor
capacitor is shown in figure 2.

\[ C = \frac{Q}{V} \]

Capacitance of a capacitor depends on the geometry of the conductor and the permittivity of the medium between them and does not depend on the charge or potential difference between conductors. The capacitance can be computed by assuming the same time \(-Q\) on the other conductor, first determining \(\overrightarrow{E}\) using Gauss determining \(V = -\int\overrightarrow{E}.d\overrightarrow{l}\). We illustrate this procedure by taking the example of a para

**Parallel plate capacitor**
For the parallel plate capacitor shown in the figure, let each plate has area \( A \) and a distance \( h \) separates the plates. A dielectric of permittivity \( \varepsilon \) fills the region between the plates. The electric field lines are confined between the plates. We ignore the flux fringing at the edges of the plates and charges are assumed to be uniformly distributed over the conducting plates with densities \( \rho_1 \) and \( -\rho_2 \), \( \rho_1 = \frac{Q}{A} \).

By Gauss’s theorem we can write,

\[
E = \frac{\rho_1}{\varepsilon} = \frac{Q}{\varepsilon A}
\]

The same approach may be extended to more than two capacitors connected in series.

**Parallel Case:** For the parallel case, the voltages across the capacitors are the same.

The total charge \( Q = Q_1 + Q_2 = C_1V + C_2V \)

Determine the energy stored and energy density in a static electric field?
We have stated that the electric potential at a point in an electric field is the amount
of work required to bring a unit positive charge from infinity (reference of zero potential) to that point. To determine the energy that is present in an assembly of charges, let us first determine the amount of work required to assemble them. Let us consider a number of discrete charges \( Q_1, Q_2, \ldots, Q_N \) are brought from infinity to their present position one by one. Since initially there is no field present, the amount of work done in brings \( Q_1 \) is zero. \( Q_2 \) is brought in the presence of the field of \( Q_1 \), the work done \( W_1 = Q_2 V_{21} \) where \( V_{21} \) is the potential at the location of \( Q_2 \) due to \( Q_1 \).

Proceeding in this manner, we can write, the total work done

\[
W = \sum_{i=1}^{N} \left( Q_i V_{i1} + \sum_{j=1}^{i} Q_j V_{i j} \right)
\]

Had the charges been brought in the reverse order,

\[
W = \sum_{i=1}^{N} \left( Q_i V_{i1} + \sum_{j=1}^{i} Q_j V_{i j} \right) + \sum_{i=1}^{N} \left( Q_i V_{i N} + \sum_{j=1}^{N} Q_j V_{i j} \right) - (N-1)Q_{i1}V_{i1}
\]

Therefore,

\[
2W = (Q_1 V_{11} + \sum_{i=1}^{2} Q_i V_{i1}) + (Q_2 V_{21} + \sum_{i=1}^{2} Q_i V_{i2}) + (Q_3 V_{31} + \sum_{i=1}^{3} Q_i V_{i3}) + \cdots
\]

Here \( V_{IJ} \) represent voltage at the \( I^{th} \) charge location due to \( J^{th} \) charge. Therefore,

\[
2W = V_1 Q_1 + \sum_{i=2}^{N} Q_i V_{i1} = \sum_{i=1}^{N} V_i Q_i
\]

Or,

\[
W = \frac{1}{2} \sum_{i=1}^{N} V_i Q_i
\]

If instead of discrete charges, we now have a distribution of charges over a volume \( V \) then we can write,

\[
W = \frac{1}{2} \int_V \rho \cdot d\mathbf{v}
\]

Where \( \rho \) the volume charge density and \( V \) represents the potential function.

Since, \( \rho = \nabla \cdot \mathbf{D} \), we can write

\[
W = \frac{1}{2} \int_V (\nabla \cdot \mathbf{D}) V dv
\]

Using the vector identity,

\[
\nabla \cdot (V \mathbf{D}) = \mathbf{D} \cdot \nabla V + V \nabla \cdot \mathbf{D}
\]

we can write

\[
W = \frac{1}{2} \int_V (\nabla \cdot (V \mathbf{D})) dv - \frac{1}{2} \int_V V \nabla \cdot \mathbf{D} dv
\]

\[
= \frac{1}{2} \int_S (V \mathbf{D}) \cdot d\mathbf{s} - \frac{1}{2} \int_V (\mathbf{D} \cdot \nabla V) dv
\]

In the expression \( \int_S \), for point charges, since \( V \) varies as \( \frac{1}{r} \) and \( \mathbf{D} \) varies
\[ \frac{1}{r^2}, \text{ the term } \frac{1}{V} \text{ varies as } \frac{1}{r^2} \text{ while the area varies as } r^2. \] Hence the integral term varies at least as \( r \) and the as surface becomes large (i.e. \( r \to \infty \)) the integral term tends to zero.

Thus the equation for \( W \) reduces to

\[ W = -\frac{1}{2} \int (\overrightarrow{D} \cdot \nabla \varphi) d\varphi = \frac{1}{2} \int (\overrightarrow{D} \cdot \overrightarrow{E}) d\varphi = \frac{1}{2} \int (\epsilon \overrightarrow{E}^2) d\varphi = \int \varphi d\varphi \]

\[ \varphi = \frac{1}{2} \epsilon \overrightarrow{E}^2 \]

is called the energy density in the electrostatic field.

Explain the Electric field inside a dielectric material?

Let's consider some special examples then.

If it's known that there are no other bound charges except those sticking to the free charges, then the field will be weakened, like in the case of a dielectric-filled capacitor.

If the surface bound charges are very far away, then they can be ignored and we can say the field will be weakened. Like in the case of extended dielectric media.

If the bound charge distribution has symmetry, then we may conclude the answer easily. For example, in the case of a dielectric sphere, since the surface bound charges are spherically symmetric, their field inside is zero and their field outside cancels exactly of those of the volume bound charges. So one can conclude that when you introduce the dielectric sphere, the field inside is weakened, while the field outside remains the same.

(For completely filled capacitors)

\[ Q = CV \]

So,

\[ C = \frac{Q}{V} \]

So, \( C \) is charge stored per unit Potential Difference applied.

Now,

\[ V = Ed, \]

where \( d \) is distance between plates. \( E = VdE = Vd \)

Case 1) When you apply a constant \( V \) of 1V to capacitor \( E \) across capacitor is 1Vd1Vd which is \textbf{constant} independent of capacitance of capacitor or dielectric b/w plates.

So, \( E \) in capacitor is \textbf{constant}.

Case 2) You disconnect battery after applying a PD of 1V. And then insert a capacitor. So, \( C \) becomes \( C' \).

Clearly \( Q = C' V' \) So, since \( Q \) is constant and \( C' > C^* \), \( V' < V \).

Since, \( E = VdE = V'd \), \( E \) \textbf{decrease}. 

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2. **Write short notes on polarization?**

So far in this course we have examined static field configurations of charge distributions assumed to be fixed in free space in the absence of nearby materials (solid, liquid, or gas) composed of neutral atoms and molecules. In the presence of material bodies composed of large number of charge neutral atoms (in fluid or solid states) static charge distributions giving rise to electrostatic fields can be typically found:

1. On exterior surfaces of conductors in “steady-state”,

2. In crystal lattices occupied by ionized atoms, as in depletion regions of semiconductor junctions in diodes and transistors. In this lecture we will examine these configurations and response of materials to applied electric fields.

Conductivity and static charges on conductor surfaces: • Conductivity $\sigma$ is an emergent property of materials bodies containing free charge carriers (e.g., unbound electrons, ionized atoms or molecules) which relates the applied electric field $E$ (V/m) to the electrical current density $J$ (A/m$^2$) conducted in the material via a linear

Simple physics-based models for $\sigma$ will be discussed later. For now it is sufficient to note that: – $\sigma \to \infty$ corresponds to a perfect electrical conductor 3 (PEC) for which it is necessary that $E = 0$ (in analogy with $V = 0$ across a short circuit element) independent of $J$. – $\sigma \to 0$ corresponds to a perfect insulator for which it is necessary that $J = 0$ (in analogy with $I = 0$ through an open circuit element) independent of $E$. • While (macroscopic) $E = 0$ in PEC’s unconditionally, a conductor with a finite $\sigma$ (e.g., copper or sea water) will also have $E = 0$ in “steady state” after the decay of transient currents $J$ that may be initiated within the conductor after applying an external electric field $E_0$ (see margin).
Fig: Behavior of dielectrics in static electric field: Polarization of dielectric

Here we briefly describe the behavior of dielectrics or insulators when placed in static electric field. Ideal dielectrics do not contain free charges. As we know, all material media are composed of atoms where a positively charged nucleus (diameter $\sim 10^{-15}$m) is surrounded by negatively charged electrons (electron cloud has radius $\sim 10^{-10}$m) moving around the nucleus. Molecules of dielectrics are neutral macroscopically; an externally applied field causes small displacement of the charge particles creating small electric dipoles. These induced dipole moments modify electric fields both inside and outside dielectric material.

Molecules of some dielectric materials possess permanent dipole moments even in the absence of an external applied field. Usually such molecules consist of two or more dissimilar atoms and are called polar molecules. A common example of such molecule is water molecule $\text{H}_2\text{O}$. In polar molecules the atoms do not arrange themselves to make the net dipole moment zero. However, in the absence of an external field, the molecules arrange themselves in a random manner so that net dipole moment over a volume becomes zero. Under the influence of an applied electric field, these dipoles tend to align themselves along the field as shown in figure. There are some materials that can exhibit net permanent dipole moment even in the absence of applied field. These materials are called electrets that made by heating certain waxes or plastics in the presence of electric field. The applied field aligns the polarized molecules when the material is in the heated state and they are frozen to their new position when after the temperature is brought down to its normal temperatures. Permanent polarization remains without an externally applied field.

As a measure of intensity of polarization, polarization vector $\vec{P}$ (in C/m$^2$) is
3. Explain the Energy stored and energy density in a static electric field?

We have stated that the electric potential at a point in an electric field is the amount of work required to bring a unit positive charge from infinity (reference of zero potential) to that point. To determine the energy that is present in an assembly of charges, let us first determine the amount of work required to assemble them. Let us consider a number of discrete charges Q₁, Q₂... Qₙ are brought from infinity to their present position one by one. Since initially there is no field present, the amount of work done in brings Q₁ is zero. Q₂ is brought in the presence of the field of Q₁, the work done W₁= Q₂V₂₁ where V₂₁ is the potential at the location of Q₂ due to Q₁. Proceeding in this manner, we can write, the total work done

\[ W = V₁Q₁ + (V₂₁Q₂ + V₃₁Q₃) + \ldots \ldots + (Vₙ₁Qₙ + \ldots \ldots + Vₙ₋₁₁Qₙ₋₁) \]

Had the charges been brought in the reverse order,

\[ W = (V₁₁Q₁ + \ldots \ldots + V₁₂Q₁) + \ldots \ldots + (Vₙ₋₁₁Qₙ₋₁ + Vₙ₋₂₁Qₙ₋₂) + Vₙ₁Q₁. \]

Therefore,

\[ 2W = (V₁₁Q₁ + V₁₂Q₁ + \ldots \ldots + V₁ₙQ₁) + (V₂₁Q₂ + V₂₂Q₂ + \ldots \ldots + V₂ₙQ₂) + \ldots \ldots + (Vₙ₋₁₁Qₙ₋₁ + Vₙ₋₂₁Qₙ₋₂) + Vₙ₁Q₁. \]

Therefore,

\[ \sum_{I=1}^{n} V_IQ_I \]

Here V₁₁ represent voltage at the Ith charge location due to Jth charge. Therefore,

\[ \sum_{I=1}^{n} V_IQ_I = \sum_{I=1}^{n} \frac{1}{\Delta y} \Delta y \int V_IQ_I dV \]

\[ W = \frac{1}{2} \sum_{I=1}^{n} V_IQ_I \]

Or, If instead of discrete charges, we now have a distribution of charges over a volume v then we can write,

\[ W = \frac{1}{2} \int V \rho_v dV \]
where \( \rho_r \) is the volume charge density and \( V \) represents the potential function.

Since, \( \rho_r = \nabla \cdot \vec{D} \), we can write

\[
W = \frac{1}{2} \int \left( \nabla \cdot \vec{D} \right) dV
\]

Using the vector identity,

\( \nabla \cdot (\nabla \cdot \vec{D}) = \nabla \cdot \nabla V - \nabla \cdot \vec{D} \), we can write

\[
W = \frac{1}{2} \int \left( \nabla \cdot (\nabla \cdot \vec{D}) - \nabla \cdot \vec{D} \right) dV
\]

\[
= \frac{1}{2} \int \nabla \cdot (\nabla \cdot \vec{D}) dV - \frac{1}{2} \int \nabla \cdot \vec{D} dV
\]

In the expression \( \frac{1}{2} \int \nabla \cdot (\nabla \cdot \vec{D}) dV \), for point charges, since \( V \) varies as \( r \) and \( D \) varies as \( \frac{1}{r^3} \), the term \( V \vec{D} \) varies as \( \frac{1}{r^3} \) while the area varies as \( r^2 \). Hence the integral term varies at least as \( \frac{1}{r} \) and the as surface becomes large (i.e. \( r \to \infty \)) the integral term tends to zero.

Thus the equation for \( W \) reduces to

\[
W = -\frac{1}{2} \int \nabla \cdot \vec{D} dV = \frac{1}{2} \int \nabla \cdot (\nabla \cdot \vec{D}) dV = \frac{1}{2} \int (\varepsilon \vec{E}^2) dV = \int \varepsilon_\varepsilon dV
\]

\[
\varepsilon_\varepsilon = \frac{1}{2} \varepsilon \vec{E}^2
\]

is called the energy density in the electrostatic field.

4. **Current Density and Ohm's Law in point form?**

In our earlier discussion we have mentioned that, conductors have free electrons that move randomly under thermal agitation. In the absence of an external electric field, the average thermal velocity on a microscopic scale is zero and so is the net current in the conductor. Under the influence of an applied field, additional velocity is superimposed on the random velocities. While the external field accelerates the electron in a direction opposite to it, the collision with atomic lattice however provide the frictional mechanism by which the electrons lose some of the momentum gained between the collisions. As a result, the electrons move with some average drift velocity \( \vec{v}_d \). This drift velocity can be related to the applied electric field \( \vec{E} \) by the relationship

\[
\vec{v}_d = -\frac{\sigma \tau}{m} \vec{E}
\]

Where \( \tau \) is the average time between the collisions.
The quantity \( \vec{v}_d \), i.e., the drift velocity per unit applied field is called the mobility of electrons and denoted by \( \mu_e \).

Thus \( \mu_e = \frac{e \tau}{m} \), \( e \) is the magnitude of the electronic charge and \( \vec{v}_d = -\mu_e \vec{E} \), as the electron drifts opposite to the applied field.

Let us consider a conductor under the influence of an external electric field. If \( N_e \) represents the number of electrons per unit volume, then the charge \( \Delta Q \) crossing an area \( \Delta s \) that is normal to the direction of the drift velocity is given by:

\[
\Delta Q = -N_e e \Delta s v_d \Delta t
\]

This flow of charge constitutes a current across \( \Delta s \), which is given by,

\[
\Delta I = \frac{\Delta Q}{\Delta t} = -N_e e \Delta s v_d = N_e e \Delta s \mu_e \vec{E}
\]

The conduction current density can therefore be expressed as

\[
\vec{J}_e = \frac{\Delta I}{\Delta s} = N_e e \mu_e \vec{E} = \sigma \vec{E}
\]

Where \( \sigma \) is called the conductivity. In vector form, we can write,

\[
\vec{J}_e = \sigma \vec{E}
\]

The above equation is the alternate way of expressing Ohm's law and this relationship is valid at a point. For semiconductor material, current flow is both due to electrons and holes (however in practice, it the electron which moves), we can write

\[
\sigma = (N_e \mu_e + N_h \mu_h) e
\]

\( N_h \) and \( \mu_h \) are respectively the density and mobility of holes.

The point form Ohm's law can be used to derive the form of Ohm's law used in circuit theory relating the current through a conductor to the voltage across the conductor.

Let us consider a homogeneous conductor of conductivity \( \sigma \), length \( L \) and having a constant cross section \( S \) as shown the figure. A potential difference of \( V \) is applied across the conductor.
Fig: Homogeneous Conductor
For the conductor under consideration we can write,
\[ V = EL \]
Considering the current to be uniformly distributed,
\[ I = \int_J dI = JS = \sigma ES \]
From the above two equations,
\[ \frac{E}{L} = \frac{I}{\sigma S} \]
Therefore,
\[ V = \frac{L}{\sigma S} I = \frac{\rho L}{S} I = RI \]
Where \( \rho = \frac{1}{\sigma} \) is the resistivity in \( \Omega m \) and \( R \) is the resistance in \( \Omega \). 

Derive the continuity Equation?
Let us consider a volume \( V \) bounded by a surface \( S \). A net charge \( Q \) exists within this region. If a net current \( I \) flows across the surface out of this region, from the principle of conservation of charge this current can be equated to the time rate of decrease of charge within this volume. Similarly, if a net current flows into the region, the charge in the volume must increase at a rate equal to the current. Thus we can write,
\[ \oint_S \vec{J} \cdot d\vec{S} = -\frac{d}{dt} \int_V \rho dV I = -\frac{dQ}{dt} \]
or,
Applying divergence theorem we can write,
\[ \int_V \nabla \cdot \vec{J} dV = -\int_{\partial V} \frac{\partial \rho}{\partial t} \cdot d\vec{S} \]
It may be noted that, since \( \rho \) in general may be a function of space and time, partial derivatives are used. Further, the equation holds regardless of the choice of volume \( V \), the integrands must be equal. Therefore we can write,
\[ \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \]
The equation is called the continuity equation.

Derive the dielectric boundary conditions?

We consider interfaces between two perfect (σ = 0) dielectric regions using the conservative property of field:

\[ \nabla \cdot \mathbf{D} = \frac{d}{dL} \int \mathbf{E} \cdot dL \]

Choose a contour across interface. Contour is small enough to consider field constant along its line segments that the tangential E component is continuous across interfaces, both dielectric-to-dielectric and PEC-to-dielectric. The normal D component is continuous across dielectric interfaces; it is discontinuous across PEC-to-dielectric interfaces due to the presence of free surface charge. How to interpret field maps at interfaces?

5. **Determine approximately the permittivity of a dielectric slab from the field map at the air-dielectric interface.**

   Prove that:

   \[
   \frac{\tan \alpha_2}{\tan \alpha_1} = \frac{\varepsilon_2}{\varepsilon_1}
   \]

![Diagram showing field maps at interfaces.](image)

6. **Explain the Convection current density?**

Convection current occurs in insulators or dielectrics such as liquid, vacuum and rarified gas. Convection current results from motion of electrons or ions in an insulating medium.

Since convection current doesn’t involve conductors, hence it doesn’t satisfy ohm’s law. Consider a filament where there is a flow of charge \( \rho_v \) at a velocity \( u = u_x \hat{a}_x \).
Hence the current is given as:

\[ \Delta I = \frac{\Delta Q}{\Delta t} \]

But we know \[ \Delta Q = \rho_v \Delta V \]

Hence

\[ \Delta I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v \Delta V}{\Delta t} = \rho_v \Delta S \frac{\Delta I}{\Delta t} \]

\[ = \rho_v \Delta S \, u_y \]

Again, we also know that \[ J_y = \frac{\Delta I}{\Delta S} \]

Hence

\[ J_y = \frac{\Delta I}{\Delta S} = \rho_v \, u_y \]

Where \( u_y \) is the velocity of the moving electron or ion and \( \rho_v \) is the free volume charge density. Hence the convection current density in general is given as:

\[ J = \rho_v \, u \]

7. Explain the conduction current density?

Conduction current occurs in conductors where there are a large number of free electrons.

The net effect is that the electrons moves or drifts with an average velocity called the drift velocity \( (u_d) \) which is proportional to the applied electric field \( (E) \). Hence
according to Newton’s law, if an electron with a mass m is moving in an electric field E with an average drift velocity $v_d$, the average change in momentum of the free electron must be equal to the applied force ($F = -eE$).

\[
\frac{m v_d}{\tau} = -eE
\]

*where \( \tau \) is the average time interval between collision.*

\[
v_d = \left( -\frac{e}{m} \tau \right)E
\]

The drift velocity per unit applied electric field is called the mobility of electrons ($\mu_e$).

\[
v_d = -\mu_e E
\]

Where $\mu_e$ is defined as:

\[
\mu_e = \left( -\frac{e}{m} \right)
\]

Consider a conducting wire in which charges subjected to an electric field are moving with drift velocity $v_d$. Say there are $N_e$ free electrons per cubic meter of conductor, then the free volume charge density ($\rho_v$) within the wire is

\[
\rho_v = -e N_e
\]

the charge $\Delta Q$ is given as: $\Delta Q = \rho_v \Delta V = -e N_e \Delta S \Delta l = -e N_e \Delta S v_d \Delta t$

\[
\Delta I = \frac{\Delta Q}{\Delta t} = -N_e e \Delta S v_d
\]

*Now since \( v_d = -\mu_e E \)*

Therefore

\[
\Delta I = N_e e \Delta S \mu_e E
\]

The incremental current is thus given as: $\Delta I = N_e e \Delta S \mu_e E$

- The conduction current density is thus defined as:

\[
J_c = \frac{\Delta I}{\Delta S} = N_e e \mu_e E = \sigma E
\]
Where $\sigma$ is the conductivity of the material.

The above equation is known as the Ohm’s law in point form and is valid at every point in space. In a semiconductor, current flow is due to the movement of both electrons and holes, hence conductivity is given as:

$$\sigma = (N_e \mu_e + N_h \mu_h) e$$

UNIT-III
MAGNETO STATICS

1. **State and Explain the biot - Savart’s law?**

This law relates the magnetic field intensity $dH$ produced at a point due to a differential current element $ldl$ as shown in Fig.
Fig: Magnetic field intensity due to a current element

The magnetic field intensity \( d\vec{H} \) at \( P \) can be written

\[
d\vec{H} = \frac{I d\vec{l} \times \hat{R}}{4\pi R^2} = \frac{I d\vec{l} \times \vec{R}}{4\pi R^5}
\]

where \( R = |\vec{R}| \) is the distance of the current element from the point \( P \).

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig.

By denoting the surface current density as \( K \) (in amp/m) and volume current density as \( J \) (in amp/m²) we can write:

\[I d\vec{l} = K ds = J dv\]
(It may be noted that \( I = K d \omega = J d \alpha \))

Employing Biot-Savart Law, we can now express the magnetic field intensity \( \vec{H} \). In terms of these current distributions, we have:

\[
\vec{H} = \frac{K d \vec{\omega} \times \vec{R}}{4\pi R^3} \quad \text{for line current}
\]

\[
\vec{H} = \frac{J d \vec{\alpha} \times \vec{R}}{4\pi R^3} \quad \text{for surface current}
\]

\[
\vec{H} = \frac{J d \vec{\omega} \times \vec{R}}{4\pi R^3} \quad \text{for volume current}
\]

For volume current, suggestions, queries, and comments in the comment section below.

**2. State the Ampere’s circuital law and explain its applications?**

Ampere’s Circuit Law:

Ampere’s circuital law states that the line integral of the magnetic field \( \vec{H} \) (circulation of \( \vec{H} \)) around a closed path is the net current enclosed by this path. Mathematically,

\[
\oint \vec{H} \cdot d\vec{l} = I_{enc}
\]

The total current \( I_{enc} \) can be written as

\[
I_{enc} = \oint \vec{J} \cdot d\vec{s}
\]

By applying Stroke’s theorem, we can write

\[
\oint \vec{H} \cdot d\vec{l} = \int \nabla \times \vec{H} \cdot d\vec{s}
\]

\[
\therefore \int \nabla \times \vec{H} \cdot d\vec{s} = \oint \vec{J} \cdot d\vec{s}
\]

This is the Ampere’s law in the point form.

**Applications of Ampere’s law:**

We compute magnetic field due to an infinitely long thin current carrying conductor as shown in Fig. 4.5. Using Ampere’s Law, we consider the close path to be a circle of radius \( \rho \) as shown in the Fig. 4.5.
If we consider a small current element $\vec{dl}(= l dz \hat{z})$, $d \vec{H}$ is perpendicular to the plane containing both $d \vec{l}$ and $\vec{R}(= \rho \hat{\rho})$. Therefore only component of $\vec{H}$ that will.

\[ \vec{H} = H_r \hat{r} \]

by applying amperes law we can write,

\[ \vec{H} = \frac{1}{2\pi \rho} \hat{\rho} \]

Therefore,

\[ \vec{H} = \frac{1}{2\pi \rho} \hat{\rho} \]

2. **Write a short notes on magnetic field intensity (MFI)**

The magnetic fields generated by currents and calculated from Ampere's Law or the Biot-Savart Law are characterized by the magnetic field $B$ measured in Tesla. But when the generated fields pass through magnetic materials which themselves contribute internal magnetic fields, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field quantity, usually called the "magnetic field strength" designated by $H$. It can be defined by the relationship

\[ H = \frac{B}{\mu_0} = \frac{B}{\mu_0} - M \]

And has the value of unambiguously designating the driving magnetic influence from external currents in a material, independent of the material's magnetic response. The relationship for $B$ can be written in the equivalent form

\[ B = \mu_0(H + M) \]

$H$ and $M$ will have the same units, amperes/meter. To further distinguish $B$ from $H$, $B$ is sometimes called the magnetic flux density or the magnetic induction. The quantity $M$ in these relationships is called the magnetization of the material.

Another commonly used form for the relationship between $B$ and $H$ is

\[ B = \mu_m H \]
where

\[ \mu = \mu_m = K_m \mu_0 \]

\( \mu_0 \) being the magnetic permeability of space and \( K_m \) the relative permeability of the material. If the material does not respond to the external magnetic field by producing any magnetization, then \( K_m = 1 \). Another commonly used magnetic quantity is the magnetic susceptibility which specifies how much the relative permeability differs from one.

Magnetic susceptibility \( \chi_m = K_m - 1 \)

For paramagnetic and diamagnetic materials the relative permeability is very close to 1 and the magnetic susceptibility very close to zero. For ferromagnetic materials, these quantities may be very large. The unit for the magnetic field strength \( H \) can be derived from its relationship to the magnetic field \( B \), \( B = \mu H \). Since the unit of magnetic permeability \( \mu \) is \( \text{N/A}^2 \), then the unit for the magnetic field strength is: \( \text{T/(N/A}^2) = (\text{N/Am})/(\text{N/A}^2) = \text{A/m} \)

An older unit for magnetic field strength is the oersted: \( 1 \text{ A/m} = 0.01257 \) oersted

3. **Explain the Maxwell’s second Equation and derive div(B)=0**

**Derivation of First Equation**

\( \text{div } D = \Delta D = p \)

“The Maxwell first equation is nothing but the differential form of Gauss law of electrostatics.”

Let us consider a surface \( S \) bounding a volume \( V \) in a dielectric medium. In a dielectric medium total charge consists of free charge. If \( p \) is the charge density of free charge at a point in a small volume element \( dV \). Then Gauss’s law can be expresses as

“The total normal electrical induction over a closed surface is equal to \( -\frac{1}{\varepsilon_0} \) times of \( 1/\varepsilon_0 \) total charge enclosed.

\[ \int_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \]

\[ \int_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \int_V \rho \, dV \]

where \( p = \) charge per unit volume

\( V = \) volume enclosed by charge.
By Gauss transformation formula

\[ \int_V \nabla \cdot E \, dv = \frac{1}{\varepsilon_0} \int_V \rho \, dv \quad \left[ \int_S A \cdot n \, ds = \int_V \nabla \cdot \mathbf{A} \right] \]

\[ \nabla \cdot E = \frac{1}{\varepsilon_0} \mathbf{P} \]
\[ \varepsilon_0 \nabla \cdot E = \mathbf{P} \]
\[ \nabla \varepsilon_0 E = \mathbf{P} \]
\[ \mathbf{P} = 0 \]
Then \( \mathbf{D} = \varepsilon_0 \mathbf{E} \) \quad \left[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \right] \]

**Derivation of Second Equation**

\[ \nabla \cdot \mathbf{B} = \Delta \cdot \mathbf{B} = 0 \]

“It is nothing but the differential form of Gauss law of magneto statics.”

Since isolated magnetic poles and magnetic currents due to them have no significance. Therefore magnetic lines of force in general are either closed curves or go off to infinity. Consequently the number of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it. It means that the flux of magnetic induction \( \mathbf{B} \) across any closed surface is always zero.

Gauss law of magneto statics states that “Total normal magnetic induction over a closed surface is equal to zero.”

i.e; \( \int_S \mathbf{B} \cdot n \, ds = 0 \)

**4. Derive the Point form of Ampere’s circuital law?**

**Ampere’s modified circuital law**

According to law the work done in carrying a unit magnetic pole once around closed arbitrary path linked with the current is expressed by

\[ \int_C \mathbf{B} \cdot dl = \mu_0 \mathbf{I} \]

\( i = \text{current enclosed by the path} \)

\[ \int_C \mathbf{B} \cdot dl = \mu_0 \int_S \mathbf{J} \cdot n \, ds \]

On applying Stoke’s transformation formula in L.H.S.

\[ \int_S \nabla \times \mathbf{B} \cdot n \, ds = \int_C \mu_0 \mathbf{J} \cdot n \, ds \]

\[ \Rightarrow \int_S \left( \nabla \times \mathbf{B} - \mu_0 \mathbf{J} \right) \cdot n \, ds = 0 \]

For the validity this equation

\[ \nabla \times \mathbf{B} - \mu_0 \mathbf{J} = 0 \]

\[ \nabla \times \mu_0 \mathbf{J} \]

It is known as the fourth equation of Maxwell.

Taking divergence of both sides

\[ \text{Div} \cdot (\nabla \times \mathbf{B}) = \text{Div} \cdot (\mu_0 \mathbf{J}) \]

\[ = 0 \Rightarrow \text{Div} \cdot (\mu_0 \mathbf{J}) \]

\[ = \mu_0 \text{Div} \cdot \mathbf{J} \left[ \text{Div} \cdot (\nabla \times \mathbf{A}) = 0 \right] \]
Div \( J = 0 \)
Which means that the current is always closed and there are no source and sink? Thus we arrive at contradiction equation (3) is also in conflict with the equation of discontinuity.
But the according to law of continuity
Div \( J = \frac{d p}{d t} \)
So this equation fails and it need of little modification. So Maxwell assume that
curl \( B = \mu_0 (\text{div } J ) + \mu_0 (\text{div } J_d) \)
0 = \( \mu_0 (\text{div } J ) + \mu_0 (\text{div } J_d) \)
By putting \( \text{div } J_d = \frac{d p}{d t} \)
Div \( J_d = \text{div } dD/dt \)
\( J_d = dD/dt \) (By Maxwell first equation, div \( D = p \) in equation (4))
Putting in equation (4), we get
curl \( \mu_0 = H \) = \( \mu_0 ( J + D d/dt) \)
curl \( H = j dD/dt \)

5. Derive the Carrying wire relation between magnetic flux, magnetic flux density and MFI?

Magnetic Flux Density: In simple matter, the magnetic flux density \( \overrightarrow{B} \) related to the magnetic field intensity \( \overrightarrow{H} \) as \( \overrightarrow{B} = \mu \overrightarrow{H} \) where \( \mu \) called the permeability. In particular when we consider the free space \( \overrightarrow{B} = \mu_0 \overrightarrow{H} \) where \( \mu_0 = 4\pi \times 10^{-7} \) H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m².

The magnetic flux density through a surface is given by:
\[
\psi = \int \overrightarrow{B} \cdot d\overrightarrow{S} \quad \text{Wb}
\]
In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i.e., pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig.

This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.
6. **Explain the magnetic field due to an infinite thin current carrying conductor**

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig.

From our discussions above, it is evident that for magnetic field,

\[ \oint \mathbf{B} \cdot d\mathbf{s} = 0 \]

By applying divergence theorem, we can write:

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \int \nabla \cdot \mathbf{B} \, d\mathbf{v} = 0 \]

Hence,

\[ \nabla \cdot \mathbf{B} = 0 \]

Which is the Gauss's law for the magnetic field in point form.

Recognize MFI due to an infinite sheet of current and a long current carrying filament

Apply the MFI due to an infinite sheet of current and a long current carrying filament
Magnetic field due to an infinite thin current carrying conductor

We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current $I$ and outer conductor carrying current $-I$ as shown in figure 4.6. We compute the magnetic field as a function of $\rho$ as follows:

In the region $0 \leq \rho \leq R_1$

$$I_{enc} = I \frac{\rho^2}{R_1^2}$$

$$H_\phi = \frac{I_{enc}}{2\pi \rho} = \frac{I\rho}{2\pi R_1^2}$$

In the region $R_1 \leq \rho \leq R_2$

$$I_{enc} = I$$

$$H_\phi = \frac{I}{2\pi \rho}$$

7. Derive the Maxwell’s second equation?

Derivation of First Equation

$$\text{div } D = \Delta.D = p$$

“The Maxwell first equation is nothing but the differential form of Gauss law of electrostatics.”
Let us consider a surface S bounding a volume V in a dielectric medium. In a dielectric medium total charge consists of free charge. If \( p \) is the charge density of free charge at a point in a small volume element \( dV \). Then Gauss’s law can be expresses as

“The total normal electrical induction over a closed surface is equal to \(-\frac{1}{\varepsilon_0}\) times of total charge enclosed.

\[
\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0},
\]

\[
\int_V \mathbf{E} \cdot d\mathbf{V} = \frac{1}{\varepsilon_0} \int_V \rho \, dV
\]

Where \( p \) = charge per unit volume

\( V \) = volume enclosed by charge.

By Gauss transformation formula

\[
\int_V \nabla \cdot \mathbf{E} \, dV = \frac{1}{\varepsilon_0} \int_V \rho \, dV \quad \text{[\( \oint_S \mathbf{A} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{A} \, dV \)]}
\]

\[
\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho
\]

\[
\varepsilon_0 \nabla \cdot \mathbf{E} = \rho
\]

\[
\nabla \cdot \varepsilon_0 \mathbf{E} = \rho
\]

\[
\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho
\]

\[
\varepsilon_0 \nabla \cdot \mathbf{E} = \rho
\]

\[
\varepsilon_0 \nabla \cdot \mathbf{E} = \rho
\]

\[
P=0 \quad \text{Then} \quad \mathbf{D} = \varepsilon_0 \mathbf{E} \quad \text{[\( \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \)]}
\]

**Derivation of Second Equation**

\[
\nabla \cdot \mathbf{B} = \Delta \cdot \mathbf{B} = 0
\]

“It is nothing but the differential form of Gauss law of magneto statics.”

Since isolated magnetic poles and magnetic currents due to them have no significance. Therefore magnetic lines of force in general are either closed curves or go off to infinity. Consequently the number of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it. It means that the flux of magnetic induction \( \mathbf{B} \) across any closed surface is always zero.

Gauss law of magneto statics states that “Total normal magnetic induction over a closed surface is equal to zero.”
i. e; \[ \int_S B \cdot n \, ds = 0 \]

10. **Apply the MFI due to a straight current carrying filament**

![Diagram of a straight current carrying filament with magnetic field lines](image)

**Magnetic field due to an infinite thin current carrying conductor**

We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current $I$ and outer conductor carrying current $-I$ as shown in figure 4.6. We compute the magnetic field as a function of $\rho$ as follows:

In the region $0 \leq \rho \leq R_1$

\[ I_{enc} = I \frac{\rho^2}{R_1^2} \]

\[ B = \frac{I_{enc}}{2 \pi \rho} = \frac{I}{2 \pi R_1^2} \]

In the region $R_1 \leq \rho \leq R_2$
UNIT 4
FORCE IN MAGNETIC FIELDS

1. Explain the force on a current element in a magnetic field?

The force on a charged particle moving through a steady magnetic field may be written as the differential force exerted on a differential element of charge,

\[ dF = dQ \times B \]

Physically, the differential element of charge consists of a large number of very small, discrete charges occupying a volume which, although small, is much larger than the average separation between the charges. The differential force expressed by (4) is thus merely the sum of the forces on the individual charges. This sum, or resultant force, is not a force applied to a single object. In an analogous way, we might consider the differential gravitational force experienced by a small volume taken in a shower of falling sand. The small volume contains a large number of sand grains, and the differential force is the sum of the forces on the individual grains within the small volume. If our charges are electrons in motion in a conductor, however, we can show that the force is transferred to the conductor and that the sum of this extremely large number of extremely small forces is of practical importance.

Within the conductor, electrons are in motion throughout a region of immobile positive ions which forma crystalline array, giving the conductor its solid properties. A magnetic field which exerts forces on the electrons tends to cause them to shift position slightly and produces a small displacement between the centers of “gravity” of the positive and
negative charges.

The Coulomb forces between electrons and positive ions, however, tend to resist such a displacement. Any attempt to move the electrons, therefore, results in an attractive force between electrons and the positive ions of the crystalline lattice.

The magnetic force is thus transferred to the crystalline lattice, or to the conductor itself. The Coulomb forces are so much greater than the magnetic forces in good conductors that the actual displacement of the electrons is almost immeasurable. The charge separation that does result, however, is disclosed by the presence of a slight potential difference across the conductor sample in a direction perpendicular to both the magnetic field and the velocity of the charges. The voltage is known as the Hall voltage, and the effect itself is called the Hall effect.

Figure illustrates the direction of the Hall voltage for both positive and negative charges in motion. In Figure 8.1a, v is in the −ax direction, v × B is in the ay direction, and Q is positive, causing FQ to be in the ay direction; thus, the positive charges move to the right. In Figure b, v is now in the +ax direction, B is still in the az direction, v×B is in the −ay direction, and Q is negative; thus, FQ is again in the ay direction. Hence, the negative charges end up at the right edge. Equal currents provided by holes and electrons in semiconductors can therefore be differentiated by their Hall voltages. This is one method of determining whether a given semiconductor is n-type or p-type. Devices employ the Hall effect to measure the magnetic flux density and, in some applications where the current through the device can be made proportional to the magnetic field across it, to serve as electronic watt meters, squaring elements, and so forth.

We defined convection current density in terms of the velocity of the volume charge density,

\[ J = \rho v \]
The differential element of charge in (4) may also be expressed in terms of volume charge density, 1

\[ dQ = \rho v \, dv \]

Thus

\[ dF = \rho v \, dv \times B \]

or

\[ dF = J \times B \, dv \]

that \( J \, dv \) may be interpreted as a differential current element; that is,

\[ J \, dv = K \, dS = I \, dL \]

and thus the Lorentz force equation may be applied to surface current density,

\[ dF = K \times B \, dS \]

or to a differential current filament,

\[ dF = I \, dL \times B \]

2. Derive the differential current loop as a magnetic dipole?

Consider the application of a vertically upward force at the end of a horizontal crank handle on an elderly automobile. This cannot be the only applied force, for if it were, the entire handle would be accelerated in an upward direction. A second force, equal in magnitude to that exerted at the end of the handle, is applied in a downward direction by the bearing surface at the axis of rotation. For a 40-N force on a crank handle 0.3 m in length, the torque is 12 N-m. This figure is obtained regardless of whether the origin is considered to be on the axis of rotation (leading to 12 N-m plus 0 N - m), at the midpoint of the handle (leading to 6 N-m plus 6 N- m), or at some point not even on the handle or an extension of the handle.

We may therefore choose the most convenient origin, and this is usually on the axis of rotation and in the plane containing the applied forces if the several forces are coplanar.

With this introduction to the concept of torque, let us now consider the torque on a differential current loop in a magnetic field \( B \). The loop lies in the \( x \, y \) plane); the sides of the loop are parallel to the \( x \) and \( y \) axes and are of length \( dx \) and \( dy \). The value of the magnetic field at the center of the loop is taken as \( B_0 \).
A differential current loop in a magnetic field $B$. The torque on the loop is $\text{d} T = I \left( \text{d}x \times \text{d}y \right) \times B_0 = I \text{d}S \times B$.

$$\text{d}T = I \text{d}S \times B$$

where $\text{d}S$ is the vector area of the differential current loop and the subscript on $B_0$ has been dropped.

We now define the product of the loop current and the vector area of the loop as the differential magnetic dipole moment $\text{d}m$, with units of $\text{A} \cdot \text{m}^2$. Thus

$$\text{d}m = I \text{d}S$$

3. Torque on a current loop placed in a magnetic field  Scalar Magnetic

We have already obtained general expressions for the forces exerted on current systems. One special case is easily disposed of, for if we take our relationship for therefore on a filamentary closed circuit, as given by Eq.

$$F = -I \oint B \times dL$$

and assume a uniform magnetic flux density, then $B$ may be removed from the integral

$$F = -IB \times \oint dL$$

However, we discovered during our investigation of closed line integrals in an electrostatic potential field that
dL = 0, and therefore the force on a closed filamentary circuit in a uniform magnetic field is zero.

If the field is not uniform, the total force need not be zero.

This result for uniform fields does not have to be restricted to filamentary circuits only. The circuit may contain surface currents or volume current density as well. If the total current is divided into filaments, the force on each one is zero, as we have shown, and the total force is again zero. Therefore, any real closed circuit carrying direct currents experiences a total vector force of zero in a uniform magnetic field.

Although the force is zero, the torque is generally not equal to zero. In defining the torque, or moment, of a force, it is necessary to consider both an origin at or about which the torque is to be calculated, and the point at which the force is applied. In Figure 8.5a, we apply a force F at point P, and we establish an origin at O with a rigid lever arm R extending from O to P. The torque about point O is a vector whose magnitude is the product of the magnitudes of R, of F, and of the sine of the angle between these two vectors. The direction of the vector torque T is normal to both the force F and the lever arm R and is in the direction of progress of a right-handed screw as the lever arm is rotated into the force vector through the smaller angle. The torque is expressible as a cross product.

4. Explain the vector magnetic potential and its properties vector magnetic potential due to simple configurations?

In studying electric field problems, we introduced the concept of electric potential and computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write: From Ampere’s law, we have

\[ \nabla \times \vec{H} = \vec{J} \]

Therefore,

\[ \nabla \times (-\nabla V_m) = \vec{J} \]

But using vector identity, \( \nabla \times (\nabla V) = 0 \) we find that \( \vec{H} = -\nabla V_m \) is valid only where \( \vec{J} = 0 \). Moreover, \( V_m \) is a scalar magnetic potential is defined only in the region where \( \vec{J} = 0 \). This point can be illustrated as follows. Let us consider the cross section of a coaxial line

\[ \vec{H} = \frac{1}{2\pi\rho} \hat{\phi} \]

In the region \( a < \rho < b \), \( \vec{J} = 0 \) and
Cross Section of a Coaxial Line

If $V_m$ is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho} \frac{\partial V_m}{\partial \phi}$$

$$= \frac{I}{2\pi \rho}$$

$$\therefore V_m = -\frac{I}{2\pi} \phi + c$$

$$V_m = -\frac{I}{2\pi} \phi$$

If we set $V_m = 0$ at $\phi = 0$ then $c = 0$ and

We observe that as we make a complete lap around the current carrying conductor, we reach $\phi_0$ again but $V_m$ this time becomes

$$V_m = -\frac{I}{2\pi} (\phi_0 + 2\pi)$$

We observe that value of $V_m$ keeps changing as we complete additional laps to pass through the same point. We introduced $V_m$ analogous to electrostatic potential $V$. But for static electric fields, $\nabla \times \vec{E} = 0$ and $\oint \vec{E} \cdot d\vec{l} = 0$, whereas for steady magnetic field $\nabla \times \vec{H} = 0$ wherever $\vec{j} = 0$ but $\oint \vec{H} \cdot d\vec{l} = I$ even if $\vec{j} = 0$ along the path of integration.

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi} \phi_0$$

We now introduce the vector magnetic potential which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems. Since $\nabla \vec{B} = 0$
and we have the vector identity that for any vector $\mathbf{A}$, $\nabla \cdot (\nabla \times \mathbf{A}) = 0$, we can write $\mathbf{B} = \nabla \times \mathbf{A}$. Here, the vector field $\mathbf{A}$ is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find $\mathbf{A}$ of a given current distribution, $\mathbf{B}$ can be found from $\mathbf{A}$ through a curl operation.

We have introduced the vector function $\mathbf{A}$ and related its curl to $\mathbf{B}$. A vector function is defined fully in terms of its curl as well as divergence. The choice of $\nabla \mathbf{A}$ is made as follows.

$$\nabla \times \nabla \times \mathbf{A} = \mu \nabla \times \mathbf{H} = \mu \mathbf{J}.$$

5. **Explain the Self and Mutual inductance?**

A current through a conductor produces a magnetic field surround it. The strength of this field depends upon the value of current passing through the conductor. The direction of the magnetic field is found using the right hand grip rule, which shown. The flux pattern for this magnetic field would be number of concentric circle perpendicular to the detection of current.

Now if we wound the conductor in form of a coil or solenoid, it can be assumed that there will be concentric circular flux lines for each individual turn of the coil as shown. But it is not possible practically, as if concentric circular flux lines for each individual turn exist, they will intersect each other. However, since lines of flux cannot intersect, the flux lines for individual turn will distort to form complete flux loops around the whole coil as shown. This flux pattern of a current carrying coil is similar to a flux pattern of a bar magnet as shown.

**Self Inductance**

Whenever, current flows through a circuit or coil, flux is produced surround it and this flux also links with the coil itself. Self induced emf in a coil is produced due to its own changing flux and changing flux is caused by changing current in the coil. So, it can be concluded that self-induced emf is ultimately due to changing current in the coil itself. And self inductance is the property of a coil or solenoid, which causes a self-induced emf to be produced, when the current through it changes.

Whenever changing flux, links with a circuit, an emf is induced in the circuit. This is Faraday’s laws of electromagnetic induction. According to this law,

$$e = -N \frac{d\phi}{dt} \quad \cdots \cdots \cdots \cdots (1)$$

**Mutual Inductance**

Mutual inductance may be defined as the ability of one circuit to produce an emf in a nearby circuit by induction when current in the first circuit changes. In reverse way second circuit can also induce emf in the first circuit if current in the second circuit changes.
Coefficient of Mutual Inductance

Let’s consider two nearby coils of turns $N_1$ and $N_2$ respectively. Let us again consider, current $i_1$ flowing through first coil produces $\phi_1$. If this whole of the flux links with second coil, the weber-turn in the second coil would be $N_2\phi_1$ due to current $i_1$ in the first coil. From this, it can be said, $(N_2\phi_1)/i_1$ is the weber-turn of the second coil due to unit current in the first coil. This term is defined as co-efficient of mutual inductance. That means, mutual inductance between two coils or circuits is defined as the weber-turns in one coil or circuit due to 1 A current in the other coil or circuit.

Formula or Equation of Mutual Inductance

Now we have already found that, mutual inductance due to current in first coil is,

$$M = \frac{N_2\phi_1}{i_1}$$

6. **Calculate the Mutual inductance between a straight long wire?**

Mutual Inductance is the ratio between induced Electro Motive Force across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage. Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil. Mutual inductance is denoted as (M), it is called co-efficient of Mutual Induction between two coils.

**Mutual inductance** for two coils gives the same value when they are in mutual induction with each other. Induction in one coil due to its own rate of change of current is called self inductance (L), but due to rate of change of current of adjacent coil it gives mutual inductance (M).

From the above figure, first coil carries current $i_1$ and its self inductance is $L_1$. Along with its self inductance it has to face mutual induction due to rate of change of current $i_2$ in the second coil. Same case happens in the second coil also. Dot convention is used to mark the polarity of the mutual induction. Suppose two coils are placed nearby.

Now it can be written from these equations,

However, using the reciprocity theorem which combines Ampere’s law and the Biot-Savart’s law, one may show that the constants are equal. i.e. $M_{12} = M_{21} = M$. M is the mutual inductance for both coil in
1. State and explain the Faraday's laws of electromagnetic induction?

Faraday's Law of electromagnetic Induction

Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed. In terms of fields, we can say that a time varying magnetic field produces an electromotive force (emf) which causes a current in a closed circuit. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday's law. Mathematically, the induced emf can be written as

\[
\text{Emf} = -\frac{d\phi}{dt} \quad \text{Volts}
\]

Where \( \phi \) is the flux linkage over the closed path.

A non-zero \( \frac{d\phi}{dt} \) may result due to any of the following:

(a) Time changing flux linkage a stationary closed path.

(b) Relative motion between a steady flux a closed path.

(c) A combination of the above two cases.

The negative sign in equation 3 was introduced by Lenz in order to comply with the polarity of the induced emf. The negative sign implies that the induced emf...
will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of \( N \) tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

\[
\text{Emf} = -N \frac{d\phi}{dt} \text{ Volts}
\]

By defining the total flux linkage as

\[ \lambda = N \phi \]

The emf can be written as

\[ \text{Emf} = -\frac{d\lambda}{dt} \]

Continuing with equation (5.3), over a closed contour 'C' we can write

\[ \text{Emf} = \oint_C \vec{E}.d\vec{l} \]

Where \( \vec{E} \) is the induced electric field on the conductor to sustain the current.

2. **Explain the Maxwell’s equations in time varying fields?**

Maxwell’s fourth equation, \( \text{Curl (E)} = -\frac{\partial B}{\partial t} \) – Statically and Dynamically induced EMFs:

Further, total flux enclosed by the contour 'C' is given by

\[ \phi = \oint_S \vec{B}.d\vec{s} \]

Where \( S \) is the surface for which 'C' is the contour.

From and using in we can write
By applying stokes theorem

\[ \oint_C \overrightarrow{E} \cdot d\overrightarrow{l} = -\frac{\partial}{\partial t} \oint_S \overrightarrow{B} \cdot d\overrightarrow{s} \]

Therefore, we can write

\[ \nabla \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} \]

which is the Faraday's law in the point form

\[ \frac{d \phi}{dt} \]

We have said that non zero \( \frac{d \phi}{dt} \) can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf.

3. **Derive the Maxwell's fourth equation?**

or

4. **derive Curl (E)=-B/t?**

Maxwell’s fourth equation, \( \text{Curl} (E)=-\frac{B}{t} \) – Statically and Dynamically induced EMFs:

Further, total flux enclosed by the contour 'C' is given by

\[ \phi = \oint_S \overrightarrow{B} \cdot d\overrightarrow{s} \]

Where S is the surface for which 'C' is the contour.

From and using in we can write

\[ \oint_C \overrightarrow{E} \cdot d\overrightarrow{l} = -\frac{\partial}{\partial t} \oint_S \overrightarrow{B} \cdot d\overrightarrow{s} \]
By applying stokes theorem

$$\oint_s \nabla \times \vec{B} \cdot d\vec{s} = \oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

Therefore, we can write

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

which is the Faraday’s law in the point form

$$\frac{d \phi}{dt}$$

We have said that non zero \( \frac{d \phi}{dt} \) can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf.

5. **Explain modification of Maxwell’s equations for time varying fields?**

Gives the relationship among the field quantities in the static field. For time varying case, the relationship among the field vectors written as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \vec{D} = \rho$$

$$\nabla \vec{B} = 0$$

In addition, from the principle of conservation of charges we get the equation of continuity

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$
Let us apply the Ampere's Law for the Amperian loop shown in fig 5.3. \( I_{\text{enc}} = I \) is the total current passing through the loop. But if we draw a balloon shaped surface as in fig 5.3, no current passes through this surface and hence \( I_{\text{enc}} = 0 \). But for non steady currents such as this one, the concept of current enclosed by a loop is ill-defined since it depends on what surface you use. In fact Ampere's Law should also hold true for time varying case as well, then comes the idea of displacement current which will be introduced in the next few slides.

We can write for time varying case,

\[
\nabla \left( \nabla \times \vec{H} \right) = 0 = \nabla \cdot \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
= \nabla \cdot \vec{J} + \frac{\partial \vec{D}}{\partial t} \\
= \nabla \cdot \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \\
\therefore \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}
\]

The equation is valid for static as well as for time varying case.

Equation indicates that a time varying electric field will give rise to a magnetic field even in the absence of \( \vec{J} \). The term \( \frac{\partial \vec{D}}{\partial t} \) has a dimension of current densities \( \left( \text{Am}^2/\text{m}^2 \right) \) and is called the displacement current density.

Introduction of \( \frac{\partial \vec{D}}{\partial t} \) in \( \nabla \times \vec{H} \) equation is one of the major contributions of James Clerk Maxwell. The modified set of equations

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}
\]
\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]

\[ \nabla \cdot \vec{D} = \rho \]

\[ \nabla \cdot \vec{B} = 0 \]

is known as the Maxwell's equation and this set of equations apply in the time varying scenario, static fields are being a particular case \( \left( \frac{\partial}{\partial t} = 0 \right) \).

In the integral form

\[ \oint_c \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \]

\[ \oint_s \vec{H} \cdot d\vec{l} = \oint_s \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s} - \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} \]

(5.26b)

\[ \int_v \nabla \cdot \vec{D} \, dv = \oint_s \vec{D} \cdot d\vec{s} = \int_v \rho \, dv \]

\[ \oint \vec{B} \cdot d\vec{S} = 0 \]

The modification of Ampere’s law by Maxwell has led to the development of a unified electromagnetic field theory. By introducing the displacement current term, Maxwell could predict the propagation of EM waves. Existence of EM waves was later demonstrated by Hertz experimentally which led to the new era of radio communication.

Ampere’s law in the original form is valid only if any electric fields present are constant in time Maxwell modified the law to include timesaving electric fields,, Maxwell added an additional term which includes a factor called the displacement current, \( I_d \)

The displacement current is not the current in the conductor,, Conduction current will be used to refer to current carried by a wire or other conductor,
6. Express the time varying equations?

In our study of static fields so far, we have observed that static electric fields are produced by electric charges, static magnetic fields are produced by charges in motion or by steady current. Further, static electric field is a conservative field and has no curl, the static magnetic field is continuous and its divergence is zero. The fundamental relationships for static electric fields among the field quantities can be summarized as:

\[ \nabla \times \vec{E} = 0 \]

\[ \nabla \cdot \vec{D} = \rho_v \]

For a linear and isotropic medium,

\[ \vec{D} = \varepsilon \vec{E} \]

Similarly for the magneto static case

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{H} = \vec{J} \]

\[ \vec{B} = \mu \vec{H} \]

It can be seen that for static case, the electric field vectors \( \vec{E} \) and \( \vec{D} \) and magnetic field vectors \( \vec{B} \) and \( \vec{H} \) form separate pairs.

In this chapter we will consider the time varying scenario. In the time varying case we will observe that a changing magnetic field will produce a changing electric field and vice versa.

We begin our discussion with Faraday's Law of electromagnetic induction and then present the Maxwell's equations which form the foundation for the electromagnetic theory.
16. Unit wise-Question bank

UNIT-I

STATIC ELECTRIC FIELD

2-Marks Question and answers

1. Define scalar field?

A field is a system in which a particular physical function has a value at each and every point in that region. The distribution of a scalar quantity with a defined position in a space is called scalar field.

Ex: Temperature of atmosphere.

2. Define Vector field?

If a quantity which is specified in a region to define a field is a vector then the corresponding field is called vector field.

3. Define scaling of a vector?

This is nothing but, multiplication of a scalar with a vector. Such a multiplication changes the magnitude of a vector but not the direction.

4. What are co-planar vector?

The vectors which lie in the same plane are called co-planar vectors.

5. Define base vectors?

The base vectors are the unit vectors which are strictly oriented along the directions of the coordinate axes of the given coordinate system. Outflow of that field per unit volume.

3-Marks Question and answers

1. Define scalar and vector quantity?

The scalar is a quantity whose value may be represented by a single real number which may be positive or negative. E.g., temperature, mass, volume, density.

A quantity which has both a magnitude and a specified direction in space is called a vector.

e.g. force, velocity, displacement, acceleration.
2. **What is a unit vector? What is its function while representing a vector?**

A unit vector has a function to indicate the direction. Its magnitude is always unity, irrespective of the direction which it indicates and the coordinate system under consideration.

3. **Name 3 coordinate systems used in electromagnetic engineering?**
   1) Cartesian or rectangular coordinate system.
   2) Cylindrical coordinate system.
   3) Spherical coordinate system.

4. **How to represent a point in a Cartesian system?**

A point in rectangular coordinate system is located by three coordinates namely x, y and z coordinates. The point can be reached by moving from origin, the distance x in x direction then the distance y in y direction and finally z in z direction.

5. **Show how a point p represented in a spherical coordinate system.**

The point p can be defined as the intersection of three surfaces in spherical coordinate system.

- r - Constant which is a sphere with centre as origin?
- θ – Constant which is a right circular cone with apex as origin and axis as z axis
- Φ – Constant is a plane perpendicular to xy plane.

6. **State the relationship between Cartesian and spherical system?**

\[
\begin{align*}
    x &= r \sin \theta \cos \Phi \\
    y &= r \sin \theta \sin \Phi \\
    z &= r \cos \theta
\end{align*}
\]

Now r can be expressed as

\[
\begin{align*}
    x^2 + y^2 + z^2 &= r^2 \sin^2 \theta \cos^2 \Phi + r^2 \sin^2 \theta \sin^2 \Phi + r^2 \cos^2 \theta \\
    &= r^2 \sin^2 \theta [\sin^2 \Phi + \cos^2 \Phi] + r^2 \cos^2 \theta \\
    &= r^2 [\sin^2 \theta + \cos^2 \theta] \\
    &= r^2
\end{align*}
\]
1. Derive the Maxwell’s first law, $\text{div} (\mathbf{D}) = \mathbf{v}$ equations?

Equations give the relationship among the field quantities in the static field. For time varying case, the relationship among the field vectors written as

$$\nabla \times \mathbf{B} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{E} = 0$$

In addition, from the principle of conservation of charges we get the equation of continuity

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

The equation must be consistent with equation

The divergence of the vector flu density $\mathbf{A}$ is the outflow of flu from a small closed surface per unit volume as the volume shrinks to zero.

A positive divergence for any vector quantity indicates a source of that vector quantity at that point. Similarly, a negative divergence indicates a sink. Because the divergence of the water velocity above is zero, no source or sink exists. The expanding air, however, produces a positive divergence of the velocity, and each interior point may be considered a source.

Writing with our new term, we have

$$\text{div} \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

(rectangular)

This expression is again of a form that does not involve the charge density. It is the result of applying the definition of divergence to a differential volume element in rectangular coordinates.

If a differential volume unit $\rho \, d\rho \, d\phi \, dz$ in cylindrical coordinates, or $r^2 \sin \theta \, dr \, d\theta \, d\phi$ in spherical coordinates, had been chosen, expressions for divergence involving the components of the vector in the particular coordinate system and involving partial derivatives

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with respect to the variables of that system would have been obtained. These expressions are obtained in Appendix A and are given here for convenience:

\[
\text{div } \mathbf{D} = \frac{1}{\rho} \frac{\partial (\rho D)}{\partial \rho} + \frac{1}{\rho} \frac{\partial D_\varphi}{\partial \varphi} \frac{\partial D_z}{\partial z} \quad \text{(cylindrical)}
\]

\[
\text{div } \mathbf{D} = \frac{1}{r^2} \frac{\partial (r^2 D)}{\partial r} + \frac{1}{r \sin \vartheta} \frac{\partial D_\varphi}{\partial \varphi} + \frac{1}{r \sin \vartheta} \frac{\partial D_\varphi}{\partial \varphi} \quad \text{(spherical)}
\]

These relationships are also shown inside the back cover for easy reference. It should be noted that the divergence is an operation which is performed on a vector, but that the result is a scalar. We should recall that, in a somewhat similar way, the dot or scalar product was a multiplication of two vectors which yielded a scalar.

2. **Define the electric dipole & Explain EFI due to an electric dipole?**

**Electric Dipole**

An electric dipole consists of two point charges of equal magnitude but of opposite sign and separated by a small distance.

As shown in figure 2.11, the dipole is formed by the two point charges \( Q \) and \( -Q \) separated by a distance \( d \), the charges being placed symmetrically about the origin.

Let us consider a point \( P \) at a distance \( r \), where we are interested to find the field.

The potential at \( P \) due to the dipole can be written as:

\[
\mathcal{V} = \frac{1}{4 \pi \varepsilon_0} \left[ \frac{Q}{r_1} - \frac{Q}{r_2} \right] = \frac{Q}{4 \pi \varepsilon_0} \left\{ \frac{r_2 - r_1}{r_1 r_2} \right\}
\]
When $r_1$ and $r_2 >> d$, we can write

$$r_2 - r_1 = \frac{2d}{2} \cos \theta = d \cos \theta$$

and $r_1 \approx r_2 \approx r$.

Therefore,

$$V = \frac{Q \cdot d \cos \theta}{4\pi \epsilon_0 r^2}.$$

We can write,

$$Qd \cos \theta = Q \hat{d} \cdot \hat{r}.$$

The quantity $\vec{P} = Q \vec{d}$ is called the dipole moment of the electric dipole.

Hence the expression for the electric potential can now be written as:

$$V = \frac{\vec{P} \cdot \hat{r}}{4\pi \epsilon_0 r^2}.$$

It may be noted that while potential of an isolated charge varies with distance as $1/r$ that of an electric dipole varies as $1/r^2$ with distance.

If the dipole is not centered at the origin, but the dipole center lies at $\vec{r}'$, the expression for the potential can be written as:

$$V = \frac{\vec{P} \cdot (\vec{r} - \vec{r}')}{{4\pi \epsilon_0} |\vec{r} - \vec{r}'|^2}.$$

The electric field for the dipole centered at the origin can be computed as
The electric dipole moment is a measure of the separation of positive and negative electrical charges within a system, that is, a measure of the system's overall polarity. The electric field strength of the dipole is proportional to the magnitude of dipole moment. The SI units for electric dipole moment are coulomb-meter (C-m), however the most commonly used unit is the Debye (D).

Theoretically, an electric dipole is defined by the first-order term of the multiple expansions, and consists of two equal and opposite charges infinitely close together. This is unrealistic, as real dipoles have separated charge. However, because the charge separation is very small compared to everyday lengths, the error introduced by treating real dipoles like they are theoretically perfect is usually negligible. The direction of dipole is usually defined from the negative charge towards the positive charge.

\[ \vec{P} = qd \]

An object with an electric dipole moment is subject to a torque \( \tau \) when placed in an external electric field. The torque tends to align the dipole with the field. A dipole aligned parallel to an electric field has lower potential energy than a dipole making some angle with it. For a spatially uniform electric field \( \vec{E} \), the torque is given by:

\[
\vec{E} = -\nabla V = -\left( \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} \right)
= \frac{Qd \cos \theta}{2\pi \varepsilon_0 r^3} \hat{r} + \frac{Qd \sin \theta}{4\pi \varepsilon_0 r^3} \hat{\theta}
= -\frac{Qd}{4\pi \varepsilon_0 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})
\]

\[
\vec{E} = \frac{\vec{P}}{4\pi \varepsilon_0 r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})
\]
Where \( p \) is the dipole moment, and the symbol "×" refers to the vector cross product. The field vector and the dipole vector define a plane, and the torque is directed normal to that plane with the direction given by the right-hand rule.

A dipole orients co- or anti-parallel to the direction in which a non-uniform electric field is increasing (gradient of the field) will not experience a torque, only a force in the direction of its dipole moment. It can be shown that this force will always be parallel to the dipole moment regardless of co- or anti-parallel orientation of the dipole.

4. **Derive the potential gradient** \( \nabla \Phi = 0 \)?

The potential of a single point charge at the origin depends solely on the radial distance to the observation point \( A \)

\[
V_A = \frac{Q}{4\pi \varepsilon} \cdot \frac{1}{r_A}
\]

the potential difference \( V_{AB} \) between points \( A \) and \( B \) depends solely on their radial distances from the origin

\[
V_{AB} = \int_{A}^{B} \mathbf{E} \cdot d\mathbf{L} = \int_{r_A}^{r_B} \frac{Q}{4\pi \varepsilon r^2} \mathbf{a}_r \cdot \frac{d\mathbf{a}_r}{dr} dr = \frac{Q}{4\pi \varepsilon} \left( \frac{1}{r_A} - \frac{1}{r_B} \right)
\]

angular positions, \( \theta \) and \( \phi \), of observation points do not matter. path of integration does not matter – integrand has only \( r \) component and \( r \) dependence. if path of integration is closed – potential difference is zero

\[
V_{AA} = \oint_{C} \mathbf{E} \cdot d\mathbf{L} = \frac{Q}{4\pi \varepsilon} \left( \frac{1}{r_A} - \frac{1}{r_A} \right) = 0
\]
Conservative property of potential follows from superposition and conservative property of potential of point charge. If work along a closed path is zero for a single point charge, it will be zero for any collection of charges. Electrostatic potential taken on a closed integration path is zero.

\[ \nabla = \frac{\partial}{\partial x} \mathbf{a}_x + \frac{\partial}{\partial y} \mathbf{a}_y + \frac{\partial}{\partial z} \mathbf{a}_z \]

\[ \Rightarrow \mathbf{E} = -\nabla V, \quad \text{V/m} \quad \nabla V \equiv \text{grad } V \]

5. Derive the Laplace and Poisson equations?

Obtaining Poisson's equation is exceedingly simple, for from the point form of Gauss's law,

\[ \nabla \cdot \mathbf{D} = \rho_v \]  

(1)

the definition of \( \mathbf{D} \),

\[ \mathbf{D} = \varepsilon \mathbf{E} \]  

(2)

and the gradient relationship,

\[ \mathbf{E} = -\nabla V \]  

(3)

by substitution we have

\[ \nabla \cdot \mathbf{D} = \nabla \cdot (\varepsilon \mathbf{E}) = -\nabla \cdot (\varepsilon \nabla V) = \rho_v \]

or

\[ \nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon} \]  

(4)

for a homogeneous region in which \( \varepsilon \) is constant.

Equation (4) is Poisson's equation, but the "double \( \nabla \)" operation must be interpreted and expanded, at least in cartesian coordinates, before the equation can be useful. In cartesian coordinates,

\[ \nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \]

\[ \nabla V = \frac{\partial V}{\partial x} \mathbf{a}_x + \frac{\partial V}{\partial y} \mathbf{a}_y + \frac{\partial V}{\partial z} \mathbf{a}_z \]
and therefore

\[
\nabla \cdot \nabla V = \frac{\partial}{\partial x} \left( \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial V}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial V}{\partial z} \right)
\]

\[= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \tag{5}\]

Usually the operation \( \nabla \cdot \nabla \) is abbreviated \( \nabla^2 \) (and pronounced “del squared”), a good reminder of the second-order partial derivatives appearing in (5), and we have

\[
\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon} \tag{6}\]

in cartesian coordinates.

If \( \rho_v = 0 \), indicating zero volume charge density, but allowing point charges, line charge, and surface charge density to exist at singular locations as sources of the field, then

\[
\nabla^2 V = 0 \tag{7}\]

which is Laplace’s equation. The \( \nabla^2 \) operation is called the Laplacian of \( V \).

In cartesian coordinates Laplace’s equation is

\[
\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{(cartesian)} \tag{8}\]

and the form of \( \nabla^2 V \) in cylindrical and spherical coordinates may be obtained by using the expressions for the divergence and gradient already obtained in those coordinate systems. For reference, the Laplacian in cylindrical coordinates is

\[
\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \quad \text{(cylindrical)} \tag{9}\]
Objective Type Questions

1. “Coulomb law” based on
   a) Field
   b) Charge
   c) Permittivity
   d) Force
   **Answer: b**

2. Coulomb's law applied to
   a) Electrostatics
   b) Magnetostatics
   c) Electromagnetic
   d) Maxwell theory
   **Answer: a**

3. Find the force between 2C and -1C separated by a distance 1m in air (in Newton).
   a) $18 \times 10^6$
   b) $-18 \times 10^6$
   c) $18 \times 10^{-6}$
   d) $18 \times 10^{-6}$
   **Answer: b**

4. Two charges 1C and -4C exists in air. What is the direction of force?
   a) Away from 1C
   b) Away from -4C
   c) From 1C to -4C
   d) From -4C to 1C
   **Answer: C**

5. Find the force of interaction between 60 stat coulomb and 37.5 stat coulomb spaced 7.5 cm apart in transformer oil ($\varepsilon_r=2.2$) in $10^{-4}$ N,
a) 8.15
b) 5.18
c) 1.518
d) 1.815

**Answer: d**

6. Find the force between two charges when they are brought in contact and separated by 4 cm apart, charges are 2 nC and -1 nC, in μN.

a) 1.
b) 2.44
c) 1.404
d) 2.404

**Answer: c.**

7. Coulomb's law is an implication of which law?
   a) Ampere law
   b) Gauss law
   c) Biot-Savart law
   d) Lenz law

**Answer: b**

8. Two small diameter 10 gm dielectric balls can slide freely on a vertical channel. Each carry a negative charge of 1 μC. Find the separation between the balls if the lower ball is restrained from moving.

   a) 0.5
   b) 0.4
   c) 0.3
   d) 0.2

**Answer: c.**

9. A charge of 2 X 10^{-7} C is acted upon by a force of 0.1 N. Determine the distance to the other charge

   a) 4.5 X 10^{-7} C
   b) 0.05 X 10^{-7} C
   c) 0.07 X 10^{-7} C
   d) 0.09 X 10^{-7} C
Answer: d

10. If a charge Q₁, the effect of charge Q₂ on Q₁ will be,
   a) $F₁ = F₂$
   b) $F₁ = -F₂$
   c) $F₁ = F₂ = 0$
   d) $F₁$ and $F₂$ are not equal
   Answer: b

Fill In The Blanks

1. Divergence theorem is based on ___________.
2. The Gaussian surface for a line charge will be ___________.
3. The Gaussian surface for a point charge will be ___________.
4. Circular disc of radius 5m with a surface charge density $ρ_s = 10 \sin φ$ is enclosed by surface. What is the net flux crossing the surface _________________
5. The total charge of a surface with densities 1, 2, ..., 10 is _________________
6. The work done by a charge of 10μC with a potential 4.386 is (in μJ) _________________
7. The potential of a coaxial cylinder with charge density 1 unit, inner radius 1m and outer cylinder 2m is (in $10^9$) _________________
8. The potential due to a charged ring of density 2 units with radius 2m and the point at which potential is measured is at a distance of 1m from the ring _________________
9. Gauss law cannot be used to find which of the following quantity _________________
10. The electric field intensity is defined as _________________

Answers:

1. Gauss law 2. Cylinder 3. Sphere 4. 0 5. 55 6. 43.86 7. 12.47 8. $72\pi$
9. Permittivity 10. Field lines/area
UNIT II
DIELECTRICS & CAPACITANCE

2-Marks Question and answers

1. Define point charge?
A point charge means that electric charge which is separated on a surface or space whose geometrical dimensions are very small compared to other dimensions, in which the effect of electric field to be studied.

2. Define one coulomb?
One coulomb of charge is defined as the charge possessed by $(1/1.602 \times 10^{-9})$ i.e $6 \times 10^{18}$ number of electrons.

3. What are the various types of charge distribution? Give an example for each.
   1. Point charge - Ex. Positive charge

5. State the assumptions made while defining a Coulomb’s law?
   1) The two charges are stationary.
   2) The two charges are point charge.

6. What is an equipotential surface?
An equipotential surface is an imaginary surface in an electric field of a given charge distribution, in which all points on the surface are at the same electric potential.

7. What is an electric flux?
The total number of lines of force in any particular electric field is called electric flux. It is represented by the symbol $\psi$. Similar to the charge, unit of electric flux is also Coulomb.
3-Marks Question and answers

1. **What is Gaussian surface? What are the conditions to be satisfied in special Gaussian surface?**
   The surface over which is the Gauss’s law is applied is called Gaussian surface. Obviously such a surface is a closed surface and it has to satisfy the following conditions.

   1) The surface may be irregular but should be sufficiently large so as to enclose the entire charge.
   2) The surface must be closed.
   3) At each point of the surface $D$ is either normal or tangential to the surface.
   4) The electric flux density $D$ is constant over the surface at which $D$ is normal.

2. **What is Gradient of $V$? Define Absolute potential.**
   The maximum value of rate of change of potential with distance $dv/dL$ is called gradient of $V$
   The work done in moving a unit charge from infinity (or from reference point at which potential is zero) to the point under the consideration against $E$ is called absolute potential of that point.

3. **What is Polarization?**
   The applied field $E$ shifts the charges inside the dielectric to induce the electric dipoles. This process is called Polarization.

4. **Define the unit of Potential difference? Define potential difference?**
   The unit of potential difference is Volt. One Volt potential difference is one Joule of work done in moving unit charge from one point to other in the field. The work done per unit charge in moving unit charge from B to A in the field $E$ is called potential difference between the points B to A.

5. **What is method of images?**
   The replacement of the actual problem with boundaries by an enlarged region or with image charges but no boundaries is called the method of images.
5 marks questions and answers

1. **Explain the phenomenon of parallel plate capacitors?**

   Capacitance and Capacitors

   We have already stated that a conductor in an electrostatic field is an Equi potential body and any charge given to such conductor will distribute themselves in such a manner that electric field inside the conductor vanishes. If an additional amount of charge is supplied to an isolated conductor at a given potential, this additional charge will increase the surface charge density \( \rho \). Since the potential of the conductor is given by

   \[
   V = \frac{1}{4 \pi \varepsilon_0} \int \frac{\rho ds}{r},
   \]

   the potential of the conductor will also increase maintaining the ratio \( \frac{Q}{V} \) same. Thus we can write

   \[
   C = \frac{Q}{V}
   \]

   where the constant of proportionality \( C \) is called the capacitance of the isolated conductor. SI unit of capacitance is Coulomb/ Volt also called Farad denoted by \( F \). It can be seen that if \( V=1 \), \( C = Q \). Thus capacity of an isolated conductor can also be defined as the amount of charge in Coulomb required to raise the potential of the conductor by 1 Volt.

   Of considerable interest in practice is a capacitor that consists of two (or more) conductors carrying equal and opposite charges and separated by some dielectric media or free space. The conductors may have arbitrary shapes. A two-conductor capacitor is shown in figure 2.

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**Fig : Capacitance and Capacitors**
When a d-c voltage source is connected between the conductors, a charge transfer occurs which results into a positive charge on one conductor and negative charge on the other conductor. The conductors are equipotential surfaces and the field lines are perpendicular to the conductor surface. If $V$ is the mean potential difference between the conductors, the capacitance is given by $C = \frac{Q}{V}$. Capacitance of a capacitor depends on the geometry of the conductor and the permittivity of the medium between them and does not depend on the charge or potential difference between conductors. The capacitance can be computed by assuming $Q$ (at the same time $-Q$ on the other conductor), first determining $\vec{E}$ using Gauss’s theorem and then determining $V = -\int \vec{E} \cdot d\vec{l}$. We illustrate this procedure by taking the example of a parallel plate capacitor.

![Parallel Plate Capacitor](image)

For the parallel plate capacitor shown in the figure, let each plate has area $A$ and a distance $h$ separates the plates. A dielectric of permittivity $\varepsilon$ fills the region between the plates. The electric field lines are confined between the plates. We ignore the flux fringing at the edges of the plates and charges are assumed to be uniformly distributed over the conducting plates with densities $\rho_+$ and $\rho_-$, $\rho_+ = \frac{Q}{A}$. 

By Gauss’s theorem we can write,

$$E = \frac{\rho_+}{\varepsilon} = \frac{Q}{A\varepsilon}$$
Fig: Parallel Connection of Capacitors
The same approach may be extended to more than two capacitors connected in series.
Parallel Case: For the parallel case, the voltages across the capacitors are the same.
The total charge \( Q = Q_1 + Q_2 = C_1V + C_2V \)

2. Determine the energy stored and energy density in a static electric field?
We have stated that the electric potential at a point in an electric field is the amount of work required to bring a unit positive charge from infinity (reference of zero potential) to that point. To determine the energy that is present in an assembly of charges, let us first determine the amount of work required to assemble them. Let us consider a number of discrete charges \( Q_1, Q_2, \ldots, Q_N \) are brought from infinity to their present position one by one. Since initially there is no field present, the amount of work done in bringing \( Q_1 \) is zero. \( Q_2 \) is brought in the presence of the field of \( Q_1 \), the work done \( W_1 = Q_2V_{21} \) where \( V_{21} \) is the potential at the location of \( Q_2 \) due to \( Q_1 \). Proceeding in this manner, we can write, the total work done
\[
W = V_{21}Q_2 + (V_{31}Q_3 + V_{32}Q_2) + \ldots + (V_{N1}Q_N + \ldots + V_{N(N-1)}Q_{N-1})
\]
Had the charges been brought in the reverse order,
\[
W = (V_{1N}Q_1 + \ldots + V_{12}Q_2) + \ldots + (V_{N-2N}Q_{N-2} + V_{(N-2)N}Q_{N-1}) + V_{(N-1)N}Q_N
\]
Therefore,
\[
2W = (V_{1N} + V_{1(N-1)} + \ldots + V_{12} + V_{2N})Q_1 + (V_{2N} + V_{2(N-1)} + \ldots + V_{23} + V_{3N})Q_2 + \ldots + (V_{N1} + \ldots + V_{N2} + V_{N(N-1)})Q_N
\]
Here \( V_{IJ} \) represent voltage at the \( I^{th} \) charge location due to \( J^{th} \) charge. Therefore,
\[
2W = V_1Q_1 + \ldots + V_NQ_N = \sum_{I=1}^{N} V_IQ_I
\]
\[
W = \frac{1}{2} \sum_{I=1}^{N} V_IQ_I
\]
Or,
If instead of discrete charges, we now have a distribution of charges over a volume \( V \) then we can write,
\[
W = \frac{1}{2} \int \rho_v dV
\]
Where \( \rho_v \) the volume charge density and \( V \) is represents the potential function.
Since, \( \rho = \nabla \cdot \vec{D} \), we can write
\[
W = \frac{1}{2} \int \nabla \cdot (\nabla \cdot \vec{D}) \, dV
\]
Using the vector identity,
\[
\nabla \cdot (\nabla \cdot \vec{D}) = \vec{D} \cdot \nabla V + V \nabla \cdot \vec{D}
\]
we can write
\[
W = \frac{1}{2} \int \left( \nabla \cdot (\nabla \cdot \vec{D}) - \vec{D} \cdot \nabla V \right) \, dV
\]
\[
= \frac{1}{2} \int \delta (V \vec{D}) \, d\vec{s} - \frac{1}{2} \int (\vec{D} \cdot \nabla V) \, dV
\]
In the expression \( \delta (V \vec{D}) \), for point charges, since \( V \) varies as \( \frac{1}{r} \) and \( D \) varies as \( \frac{1}{r^2} \), the term \( V \vec{D} \) varies as \( \frac{1}{r^3} \) while the area varies as \( r^2 \). Hence the integral term varies at least as \( \frac{1}{r^3} \) and the area as \( r^2 \) tends to zero. Thus the equation for \( W \) reduces to
\[
W = -\frac{1}{2} \int (\vec{D} \cdot \nabla V) \, dV = \frac{1}{2} \int (\vec{D} \cdot \vec{E}) \, dV = \frac{1}{2} \int (\varepsilon \vec{E}^2) \, dV = \int \vec{w} \, dV
\]
\[
\vec{w} = \frac{1}{2} \varepsilon \vec{E}^2
\]
is called the energy density in the electrostatic field.

3. **Explain the Electric field inside a dielectric material?**

Let’s consider some special examples then.

If it’s known that there are no other bound charges except those sticking to the free charges, then the field will be weakened, like in the case of a dielectric-filled capacitor. If the surface bound charges are very far away, then they can be ignored and we can say the field will be weakened. Like in the case of extended dielectric media.

If the bound charge distribution has symmetry, then we may conclude the answer easily. For example, in the case of a dielectric sphere, since the surface bound charges are spherically symmetric, their field inside is zero and their field outside cancels exactly of those of the volume bound charges. So one can conclude that when you introduce the dielectric sphere, the field inside is weakened, while the field outside remains the same.

(For completely filled capacitors)
\[
Q = CV
\]
So,
\[
C = \frac{Q}{V}
\]
So, \( C \) is charge stored per unit Potential Difference applied.
Now,
\[ V = Ed \]
where \( d \) is distance between plates.

Case 1) When you apply a constant \( V \) of 1V to capacitor \( E \) across capacitor is \( 1Vd \) which is **constant** independent of capacitance of capacitor or dielectric b/w plates.

So, \( E \) in capacitor is **constant**.

Case 2) You disconnect battery after applying a PD of 1V. And then insert a capacitor.

So, \( C \) becomes \( C' \).

Clearly \( Q = C' V' \) So, since \( Q \) is constant and \( C' > C \), **V' < V**.

Since,
\[ E = V'd = V'd \]
\[ E \text{ decrease.} \]

4. **Write a short notes on polarization ?**

So far in this course we have examined static field configurations of charge distributions assumed to be fixed in free space in the absence of nearby materials (solid, liquid, or gas) composed of neutral atoms and molecules. In the presence of material bodies composed of large number of charge neutral atoms (in fluid or solid states) static charge distributions giving rise to electrostatic fields can be typically found:

1. On exterior surfaces of conductors in “steady-state”,

2. In crystal lattices occupied by ionized atoms, as in depletion regions of semiconductor junctions in diodes and transistors. In this lecture we will examine these configurations and response of materials to applied electric fields.

Conductivity and static charges on conductor surfaces: • Conductivity \( \sigma \) is an emergent property of materials bodies containing free charge carriers (e.g., unbound electrons, ionized atoms or molecules) which relates the applied electric field \( E \) (V/m) to the electrical current density \( J \) (A/m²) conducted in the material via a linear relation:

\[ \sigma \rightarrow \infty \text{ corresponds to a perfect electrical conductor (PEC) for which it is necessary that } E = 0 \text{ (in analogy with } V = 0 \text{ across a short circuit element) independent of } J. \]

\[ \sigma \rightarrow 0 \text{ corresponds to a perfect insulator for which it is necessary that } J = 0 \text{ (in analogy with } I = 0 \text{ through an open circuit element) independent of } E. \]

While (macroscopic) \( E = 0 \) in PEC’s unconditionally, a conductor with a finite \( \sigma \) (e.g., copper or sea water) will also have \( E = 0 \) in “steady state” after the decay of transient currents \( J \) that may be initiated within the conductor after applying an external electric field \( E_0 \) (see margin).
Here we briefly describe the behavior of dielectrics or insulators when placed in static electric field. Ideal dielectrics do not contain free charges. As we know, all material media are composed of atoms where a positively charged nucleus (diameter \( \sim 10^{-15} \text{m} \)) is surrounded by negatively charged electrons (electron cloud has radius \( \sim 10^{-10} \text{m} \)) moving around the nucleus. Molecules of dielectrics are neutral macroscopically; an externally applied field causes small displacement of the charge particles creating small electric dipoles. These induced dipole moments modify electric fields both inside and outside dielectric material.

Molecules of some dielectric materials possess permanent dipole moments even in the absence of an external applied field. Usually such molecules consist of two or more dissimilar atoms and are called polar molecules. A common example of such molecule is water molecule \( \text{H}_2\text{O} \). In polar molecules the atoms do not arrange themselves to make the net dipole moment zero. However, in the absence of an external field, the molecules arrange themselves in a random manner so that net dipole moment over a volume becomes zero. Under the influence of an applied electric field, these dipoles tend to align themselves along the field as shown in figure. There are some materials that can exhibit net permanent dipole moment even in the absence of applied field. These materials are called electrets that made by heating certain waxes or plastics in the presence of electric field. The applied field aligns the polarized molecules when the material is in the heated state and they are frozen to their new position when after the temperature is brought down to its normal temperatures. Permanent polarization remains without an externally applied field.

As a measure of intensity of polarization, polarization vector \( \vec{P} \) (in \( \text{C/m}^2 \)) is defined as:

\[
\vec{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{k=1}^{n\Delta v} \vec{P}_k
\]

\[.......................(2.59)\]
5. Explain the Energy stored and energy density in a static electric field?

We have stated that the electric potential at a point in an electric field is the amount of work required to bring a unit positive charge from infinity (reference of zero potential) to that point. To determine the energy that is present in an assembly of charges, let us first determine the amount of work required to assemble them. Let us consider a number of discrete charges $Q_1, Q_2, ..., Q_N$ are brought from infinity to their present position one by one. Since initially there is no field present, the amount of work done in bringing $Q_1$ is zero. $Q_2$ is brought in the presence of the field of $Q_1$, the work done $W_1 = Q_2V_{21}$ where $V_{21}$ is the potential at the location of $Q_2$ due to $Q_1$. Proceeding in this manner, we can write, the total work done

$$W = V_{21}Q_2 + (V_{31}Q_3 + V_{32}Q_3) + .... + (V_{N1}Q_N + .... + V_{N(N-1)}Q_N)$$

Had the charges been brought in the reverse order,

$$W = (V_{1N}Q_1 + .... + V_{12}Q_2) + ................. + (V_{(N-1)N-1}Q_{N-2} + V_{(N-2)N-2}Q_{N-3}) + V_{(N-1)N-1}Q_{N-1}$$

Therefore,

$$2W = (V_{1N} + V_{1(N-1)})Q_1 + (V_{2N} + V_{2(N-1)})Q_2 + .... + (V_{N-1}N)Q_{N-2} + V_{N(N-1)}Q_N$$

Here $V_{IJ}$ represent voltage at the $I^{th}$ charge location due to $J^{th}$ charge. Therefore,

$$2W = V_1Q_1 + .... + V_NQ_N = \sum_{I=1}^{N} V_IQ_I$$

$$W = \frac{1}{2} \sum_{I=1}^{N} V_IQ_I$$

Or,

If instead of discrete charges, we now have a distribution of charges over a volume $\nu$ then we can write,

$$W = \frac{1}{2} \int \nu \rho_v dv$$

where $\rho_v$ is the volume charge density and $V$ represents the potential function.

Since, $\rho_v = \nabla \cdot \vec{D}$, we can write
Using the vector identity,
\[ \nabla \cdot (\nabla \times \mathbf{A}) = \nabla \cdot \mathbf{B} + \nabla \times \nabla \times \mathbf{A}, \]
we can write
\[ W = \frac{1}{2} \int (\nabla \cdot \mathbf{D}) dV \]
\[ = \frac{1}{2} \int \left( \nabla \cdot (\mathbf{V} \mathbf{D}) - \mathbf{D} \cdot \nabla \mathbf{V} \right) dV \]
\[ = \frac{1}{2} \int \Phi(\mathbf{V} \mathbf{D}) d\mathbf{s} - \frac{1}{2} \int (\mathbf{D} \cdot \nabla \mathbf{V}) dV \]

In the expression \( \frac{1}{2} \Phi(\mathbf{V} \mathbf{D}) d\mathbf{s} \), for point charges, since \( V \) varies as \( \frac{1}{r} \) and \( D \) varies as \( \frac{1}{r^2} \), the term \( V \mathbf{D} \) varies as \( \frac{1}{r^3} \) while the area varies as \( r^2 \). Hence the integral term varies at least as \( \frac{1}{r} \) and the as surface becomes large (i.e. \( r \to \infty \)) the integral term tends to zero. Thus the equation for \( W \) reduces to
\[ W = \frac{1}{2} \int (\mathbf{D} \cdot \nabla \mathbf{V}) dV = \frac{1}{2} \int (\mathbf{D} \cdot \mathbf{E}) dV = \frac{1}{2} \int (\varepsilon \mathbf{E}^2) dV = \int \mathbf{w}_e dV \]
\[ \mathbf{w}_e = \frac{1}{2} \varepsilon \mathbf{E}^2 \]

is called the energy density in the electrostatic field.

**Objective Type Questions**

1. A dielectric is always an insulator. But an insulator is not necessarily a dielectric. State True/False.
   a) True
   b) False
   **Answer:** a
   
2. Find the conductivity of a material with conduction current density 100 units and electric field of 4 units.
   a) 25
   b) 400
   c) 0.04
   d) 1600
   **Answer:** b
3. The potential difference in an open circuit is
   a) Zero
   b) Unity
   c) Infinity
   d) Circuit does not exist open

   **Answer: c**

4. Choose the best definition of dielectric loss.
   a) Absorption of electric energy by dielectric in an AC field
   b) Dissipation of electric energy by dielectric in a static field
   c) Dissipation of heat by dielectric
   d) Product of loss tangent and relative permittivity

   **Answer: a**

5. Calculate the capacitance of two parallel plates of area 2 units separated by a distance of 0.2m in air (in picofarad)

   a) 8.84
   b) 88.4
   c) 884.1
   d) 0.884

   **Answer: b**

6. Capacitance between two concentric shells of inner radius 2m and the outer radius infinitely large.

   a) 0.111nF
   b) 0.222nF
   c) 4.5nF
   d) 5.4 Nf

   **Answer: b**

7. The capacitance of a material refers to
a) Ability of the material to store magnetic field
b) Ability of the material to store electromagnetic field
c) Ability of the material to store electric field
d) Potential between two charged plates

**Answer:** c

8. DelGradient of a function is a constant. True/False
   a) True
   b) False

**Answer:** b

9. Divergence of gradient of a vector function is equivalent to
   a) Laplace operation
   b) Curl operation
   c) Double gradient operation
   d) Null vector

**Answer:** a

10. The gradient can be replaced by which of the following?
   a) Maxwell equation
   b) Volume integral
   c) Differential equation
   d) Surface integral

**Answer:** c

---

**Fill In The Blanks**

1. ________ is not an example of elemental solid dielectric?
2. Ionic non polar solid dielectrics contain more than one type of atoms but no permanent dipoles. State True/False ________
3. ________ the refractive index when the dielectric constant is 256 in air.
4. The best definition of polarisation is. ________
5. The polarisation vector of the material which has 100 dipoles per unit volume in a volume of 2 units.
6. Identify. Type of polarisation depends on temperature.
7. The polarisation vector in air when the susceptibility is 5 and electric field is 12 units.
8. In isotropic materials, of the following quantities will be independent of the direction?
9. the conductivity of a material with conduction current density 100 units and electric field of 4 units.

Answers:
1. Silicon. 2. True 3. 16 4. Electric dipole moment per unit volume 5. 200
6. Orientation 7. 60 8. Permittivity 9. 25
1. **What is Magnetic Field?**

The region around a magnet within which influence of the magnet can be experienced is called Magnetic Field.

2. **What are Magnetic Lines of Force?**

The existence of Magnetic Field can be experienced with the help of compass field. Such a field is represented by imaginary lines around the magnet which are called Magnetic Lines of Force.

3. **State Stoke Theorem?**

The line integral of $\mathbf{F}$ around a closed path $L$ is equal to the integral of curl of $\mathbf{F}$ over the open surface $S$ enclosed by the closed path $L$.

4. **Define scalar magnetic Potential?**

The scalar magnetic potential $V_m$ can be defined for source free region where $\mathbf{J}$ i.e. current density is zero.

5. **What is the fundamental difference between static electric and magnetic field lines?**

There is a fundamental difference between static electric and magnetic field lines. The tubes of electric flux originate and terminate on charges, whereas magnetic flux tubes are continuous.

3-Marks Question and answers

1. **State Biot Savart Law?**

The Biot Savart law states that, the magnetic field intensity $dH$ produced at a point $p$ due to a differential current element $IdL$ is,

- Proportional to the product of the current $I$ and differential length $dL$.
- Inversely proportional to the square of the distance $R$ between point $p$ and the element.

2. **Describe what are the sources of electric field and magnetic field?**

Stationary charges produce electric field that are constant in time, hence the term electrostatics.

Moving charges produce magnetic fields hence the term magnetostatics.

3. **Define Magnetic flux density.**

The total magnetic lines of force i.e. magnetic flux crossing a unit area in a plane at right angles to the direction of flux is called magnetic flux density. Unit $\text{Wb/m}^2$. 
4. **State Ampere’s circuital law.**
The line integral of magnetic field intensity $H$ around a closed path is exactly equal to the direct current enclosed by that path.

5. **Define Magnetic field Intensity.**
Magnetic Field intensity at any point in the magnetic field is defined as the force experienced by a unit north pole of one Weber strength, when placed at that point. Unit: N/Wb.

6. **What is rotational and irrotational vector field?**
If curl of a vector field exists then the field is called rotational. For irrotational vector field, the curl vanishes i.e. curl is zero.

7. **Give the application of Stoke’s theorem.**
The Stoke’s theorem is applicable for the open surface enclosed by the given closed path. Any volume is a closed surface and hence application of Stoke’s theorem to a closed surface which enclosed certain volume produces zero answer.

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5 marks question and answers

1. **Write a short notes on magnetic field intensity (MFI)**
The magnetic fields generated by currents and calculated from Ampere’s Law or the Biot-Savart Law are characterized by the magnetic field $B$ measured in Tesla. But when the generated fields pass through magnetic materials which themselves contribute internal magnetic fields, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field
quantity, usually called the "magnetic field strength" designated by H. It can be defined by the relationship

\[ H = B_0/\mu_0 = B/\mu_0 - M \]

And has the value of unambiguously designating the driving magnetic influence from external currents in a material, independent of the material's magnetic response. The relationship for B can be written in the equivalent form

\[ B = \mu_0(H + M) \]

H and M will have the same units, amperes/meter. To further distinguish B from H, B is sometimes called the magnetic flux density or the magnetic induction. The quantity M in these relationships is called the magnetization of the material.

Another commonly used form for the relationship between B and H is

\[ B = \mu_m H \]

where

\[ \mu = \mu_m = K_m \mu_0 \]

\( \mu_0 \) being the magnetic permeability of space and \( K_m \) the relative permeability of the material. If the material does not respond to the external magnetic field by producing any magnetization, then \( K_m = 1 \). Another commonly used magnetic quantity is the magnetic susceptibility which specifies how much the relative permeability differs from one.

Magnetic susceptibility \( \chi_m = K_m - 1 \)

For paramagnetic and diamagnetic materials the relative permeability is very close to 1 and the magnetic susceptibility very close to zero. For ferromagnetic materials, these quantities may be very large. The unit for the magnetic field strength H can be derived from its relationship to the magnetic field B, \( B = \mu H \). Since the unit of magnetic permeability \( \mu \) is \( \text{N/A}^2 \), then the unit for the magnetic field strength is: \( \text{T/(N/A}^2 \) = (N/Am)/(N/A}^2 \) = A/m

An older unit for magnetic field strength is the oersted: 1 A/m = 0.01257 oersted

2. **Explain the Maxwell’s second Equation and derive \( \text{div}(\mathbf{B}) = 0 \)**

**Derivation of First Equation**

\[ \text{div} \mathbf{D} = \Delta \mathbf{D} = \mathbf{p} \]

“The Maxwell first equation is nothing but the differential form of Gauss law of electrostatics.”
Let us consider a surface $S$ bounding a volume $V$ in a dielectric medium. In a dielectric medium total charge consists of free charge. If $p$ is the charge density of free charge at a point in a small volume element $dV$. Then Gauss’s law can be expresses as

“The total normal electrical induction over a closed surface is equal to – times of $1/ \varepsilon_0$ total charge enclosed.

\[
\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0},
\]

\[
\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \int_V \rho \, dV
\]

where $p = \text{charge per unit volume}$

$V = \text{volume enclosed by charge.}$

By Gauss transformation formula

\[
\int_V \text{div} \, \mathbf{E} \, dv = \frac{1}{\varepsilon_0} \int_V p \, dv \quad [\int_S \mathbf{A} \cdot d\mathbf{s} = \int_V \text{div} \mathbf{A} \, dv]
\]

\[
\text{div} \, \mathbf{E} = \frac{1}{\varepsilon_0} \mathbf{P}
\]

\[
\varepsilon_0 \text{div} \, \mathbf{E} = \mathbf{P}
\]

\[
\text{div} \varepsilon_0 \, \mathbf{E} = \mathbf{P}
\]

\[
P = 0 \quad \text{Then} \quad \mathbf{D} = \varepsilon_0 \, \mathbf{E} \quad [\mathbf{D} = \varepsilon_0 \, \mathbf{E} + \mathbf{P}]
\]

**Derivation of Second Equation**

\[
\text{div} \, \mathbf{B} = \Delta \mathbf{B} = 0
\]

“It is nothing but the differential form of Gauss law of magneto statics.”

Since isolated magnetic poles and magnetic currents due to them have no significance. Therefore magnetic lines of force in general are either closed curves or go off to infinity. Consequently the number of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it. It means that the flux of magnetic induction $\mathbf{B}$ across any closed surface is always zero.
Gauss law of magneto statics states that “Total normal magnetic induction over a closed surface is equal to zero.”

i.e; \( \int_S \mathbf{B} \cdot \mathbf{n} \, ds = 0 \)

3. Derive the Point form of Ampère’s circuital law

Ampère’s modified circuital law

According to law the work done in carrying a unit magnetic pole once around closed arbitrary path linked with the current is expressed by

\( \oint_C \mathbf{B} \, ds = \mu_0 \mathbf{i} \)

\( i = \) current enclosed by the path

\( \oint_C \mathbf{B} \, ds = \mu_0 \oint_C \mathbf{J} \cdot \mathbf{n} \, ds \)

On applying Stoke’s transformation formula in L.H.S.

\( \int_S \nabla \times \mathbf{B} \cdot \mathbf{n} \, ds = \int_C \mu_0 \mathbf{J} \cdot \mathbf{n} \, ds \)

\( \varepsilon \int_S (\nabla \times \mathbf{B} - \mu_0 \mathbf{J}) \cdot \mathbf{n} \, ds = 0 \)

For the validity this equation

\( \nabla \times \mathbf{B} - \mu_0 \mathbf{J} = 0 \)

\( \nabla \times \mathbf{B} - \mu_0 \mathbf{J} \)

It is known as the fourth equation of Maxwell.

Taking divergence of both sides

\( \text{Div} \cdot (\nabla \times \mathbf{B}) = \text{Div} (\mu_0 \mathbf{J}) \)

\( 0 = \text{Div} (\mu_0 \mathbf{J}) \)

\( \mu_0 \text{Div} \mathbf{J} [\text{Div} (\nabla \times \mathbf{A}) = 0] \)

\( \text{Div} \mathbf{J} = 0 \)

Which means that the current is always closed and there are no source and sink? Thus we arrive at contradiction equation (3) is also in conflict with the equation of discontinuity.
But the according to law of continuity

\[ \text{Div} \ J = - \frac{d \rho}{dt} \]

So this equation fails and it need of little modification. So Maxwell assume that

\[ \text{curl} \ B = \mu_0 (\text{div} \ J) + \mu_0 (\text{div} \ J_d) \]

\[ 0 = \mu_0 (\text{div} \ J) + \mu_0 (\text{div} \ J_d) \]

By putting \( \text{div} \ J_d = \frac{dp}{dt} \)

\[ \text{Div} \ J_d = \text{div} \frac{dD}{dt} \]

\[ \frac{dD}{dt} \] (By Maxwell first equation, \( \text{div} \ D = \rho \) in equation (4))

Putting in equation (4), we get

\[ \text{Curl} (\mu_0 = H) = \mu_0 (J + \frac{dD}{dt}) \]

\[ \text{Curl} \ H = \frac{dD}{dt} \]

4. Calculate the MFI due to circular and solenoid current?

A solenoid is a long coil of wire wrapped in many turns. When a current passes through it, it creates a nearly uniform magnetic field inside. Solenoids can convert electric current to mechanical action, and so are very commonly used as switches. The magnetic field within a solenoid depends upon the current and density of turns. In order to estimate roughly the force with which a solenoid pulls on ferromagnetic rods placed near it, one can use the change in magnetic field energy as the rod is inserted into the solenoid. The force is roughly

\[ \text{force on rod} = \frac{\text{change in magnetic field energy}}{\text{distance rod moves into solenoid}} \]

The energy density of the magnetic field depends on the strength of the field, squared, and also upon the magnetic permeability of the material it fills. Iron has a much, much larger permeability than a vacuum.

Even small solenoids can exert forces of a few Newton’s.
The field $dB$ due to a small element $dl$ of the circle, centered at $A$ has the magnitude

$$
\mu_0 \frac{Idl}{4 |AP|^2} = \mu_0 \frac{Idl}{4 (R^2 + a^2)}
$$

This field can be resolved into two components one along the axis OP, and the other (PS) perpendicular to it. The latter component is exactly cancelled by the perpendicular component (PS’) of the field due to a current and centred at $A’$. Field along OP has a magnitude

Earlier we worked out the magnetic field at the centre of a square loop (side length $w$) of current which lies in the XY plane with its centre at the origin. We can generalize this a bit by working out the field at any point on the $z$ axis. Start with the Biot-Savart law for linear currents:

$$
B(r) = \frac{\mu_0 I}{4\pi} \int \frac{dl' \times (r - r')}{|r - r'|^3}
$$

5. Derive the Carrying wire relation between magnetic flux, magnetic flux density and MFI

Magnetic Flux Density: In simple matter, the magnetic flux density $\vec{B}$ related to the magnetic field intensity $\vec{H}$ as $\vec{B} = \mu \vec{H}$ where $\mu$ called the permeability. In particular when we consider the free space $\vec{B} = \mu_0 \vec{H}$ where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m$^2$.

The magnetic flux density through a surface is given by:

$$\psi = \int \vec{B} \cdot d\vec{s}$$

Wb

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i.e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar
successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig.

This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.

(b) Subdivision of a magnet

(b) Magnetic field/ flux lines of a straight current carrying conductor

6. Explain the magnetic field due to an infinite thin current carrying conductor

Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig.

From our discussions above, it is evident that for magnetic field,

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

By applying divergence theorem, we can write:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \oint \nabla \cdot \mathbf{B} d\mathbf{v} = 0$$

Hence,

$$\nabla \cdot \mathbf{B} = 0$$

Which is the Gauss's law for the magnetic field in point form.

Recognize MFI due to an infinite sheet of current and a long current carrying filament

Apply the MFI due to an infinite sheet of current and a long current carrying filament
Magnetic field due to an infinite thin current carrying conductor

We consider the cross section of an infinitely long coaxial conductor, the inner conductor carrying a current $I$ and outer conductor carrying current $-I$ as shown in figure 4.6. We compute the magnetic field as a function of $\rho$ as follows:

In the region $0 \leq \rho \leq R_1$

\[ I_{enc} = \frac{I \rho^2}{R_1^2} \]

\[ H_\phi = \frac{I_{enc}}{2\pi \rho} = \frac{I \rho}{2\pi \rho^2} \]

In the region $R_1 \leq \rho \leq R_2$

\[ I_{enc} = I \]

\[ H_\phi = \frac{I}{2\pi \rho} \]

**Objective type Questions**

1. Biot Savart law in magnetic field is analogous to which law in electric field?
   a) Gauss law
   b) Faraday law
   c) Coulomb’s law
d) Ampere law

**Answer:** c

2. Which of the following cannot be computed using the Biot Savart law?

a) Magnetic field intensity  
b) Magnetic flux density  
c) Electric field intensity  
d) Permeability  

**Answer:** c

3. Find the magnetic field of a finite current element with 2A current and height 1/2π is

a) 1  
b) 2  
c) 1/2  
d) 1/4  

**Answer:** a

4. Calculate the magnetic field at a point on the centre of the circular conductor of radius 2m with current 8A.

a) 1  
b) 2  
c) 3  
d) 4  

**Answer:** b
5. Find the magnetic field intensity at the centre O of a square of the sides equal to 5m and carrying 10A of current.

   a) 1.2
   b) 1
   c) 1.6
   d) 1.8

   Answer: d

6. In a static magnetic field unitary poles exist. state true/false

   (a) true
   (b) false

   Answer: a

7. The magnetic field intensity will be zero inside a conductor state true/false

   (a) true
   (b) false

   Answer: a

8. The magnetic field at the center when a circular conductor of very high radius subjected to current

   a) 1
   b) ∞
   c) 0
   d) ∞

   Answer: c

Fill In The Blanks
1. The ratio of the orbital dipole moment to the orbital angular moment is given by \( \frac{-e}{2m} \)
2. Find the force that exists in an electromagnetic wave Lorentz force
3. The torque of a conductor is defined only in the case when the plane of the loop is parallel to the field
4. The angle at which the torque is minimum 90°
5. The magnetic moment and torque are related as follows \( T = BM \)
6. The relation between current density and magnetization \( J = \text{Curl}(M) \)
7. The point form of Ampere law is given by \( \text{Curl} (H) = J \)
8. The Ampere law is based on which theorem Stokes’s theorem
9. Electric field will be maximum outside the conductor and magnetic field will be maximum inside the conductor. State True/False True
10. Find the magnetic flux density of a finite length conductor of radius 12cm and current 3A in air (in 10⁻⁶ order) 5.00 × 10⁻⁶

Answers:

1. \( \frac{-e}{2m} \) 2. Lorentz force 3. The plane of the loop is parallel to the field 4.90
5. \( T = BM \) 6. \( J = \text{Curl}(M) \) 7. \( \text{Curl} (H) = J \) 8. Stokes’s theorem 9. True 10. 5.

UNIT-IV

FORCE IN MAGNETIC FIELDS AND MAGNETIC POTENTIAL

2-Marks Question and answers

1. Define Inductance.
   In general, inductance is also referred as self inductance as the flux produced by the current flowing through the coil links with the coil itself.

2. Define Mutual inductance.
   The mutual inductance between the two coils is defined as the ratio of flux linkage of one coil to the current in other coil.

3. Define Reluctance.
   Reluctance \( R \) is defined as the ratio of the magneto motive force to the total flux.

4. What is Lorentz force equation?
   Lorentz force equation relates mechanical force to the electrical force. It is given as the total force on a moving charge in the presence of both electric and magnetic fields.

5. Define Moment of force.
The Moment of a force or torque about a specified point is defined as the vector product of the moment arm \( R \) and the force \( F \). It is measured in Nm.

3-Marks Question and answers

1. **What is Magneto statics?**
The study of steady magnetic field, existing in a given space, produced due to the flow of direct current through a conductor is called Magneto statics.

2. **Define Right hand Thumb Rule and where it is used?**
Right hand Thumb Rule states that, hold the current carrying conductor in the right hand such that the thumb pointing in the direction of current and parallel to the conductor, then curled fingers point in the direction of magnetic lines of flux around it. It is used to determine the direction of Magnetic field around a conductor carrying a direct current.

3. **Define Right handed Screw Rule.**
It states that, imagine a right handed screw to be along the conductor carrying current with its axis parallel to the conductor and tip pointing in the direction of the current flow. Then the direction of Magnetic field is given by the direction in which screw must be turned so as to advance in the direction of current flow.

4. **What is called attenuation constant?**
When a wave propagates in the medium, it gets attenuated. The amplitude of the signal reduces. This is represented by attenuation constant \( \alpha \). It is measured in Neper per meter (NP/m). But practically it is expressed in decibel (dB).

5. **What is phase constant?**
When a wave propagates, phase change also takes place. Such a phase change is expressed by a phase constant \( \beta \). It is measured in radian per meter (rad/m).

5 Marks questions and answers

1. **Explain the force on a current element in a magnetic field?**
The force on a charged particle moving through a steady magnetic field may be written as the differential force exerted on a differential element of charge,

\[
dF = dQ \, v \times B
\]
Physically, the differential element of charge consists of a large number of very small, discrete charges occupying a volume which, although small, is much larger than the average separation between the charges. The differential force expressed by (4) is thus merely the sum of the forces on the individual charges. This sum, or resultant force, is not a force applied to a single object. In an analogous way, we might consider the differential gravitational force experienced by a small volume taken in a shower of falling sand. The small volume contains a large number of sand grains, and the differential force is the sum of the forces on the individual grains within the small volume. If our charges are electrons in motion in a conductor, however, we can show that the force is transferred to the conductor and that the sum of this extremely large number of extremely small forces is of practical importance.

Within the conductor, electrons are in motion throughout a region of immobile positive ions which forma crystalline array, giving the conductor its solid properties. A magnetic field which exerts forces on the electrons tends to cause them to shift position slightly and produces a small displacement between the centers of “gravity” of the positive and negative charges.

The Coulomb forces between electrons and positive ions, however, tend to resist such a displacement. Any attempt to move the electrons, therefore, results in an attractive force between electrons and the positive ions of the crystalline lattice.

The magnetic force is thus transferred to the crystalline lattice, or to the conductor itself. The Coulomb forces are so much greater than the magnetic forces in good conductors that the actual displacement of the electrons is almost immeasurable. The charge separation that does result, however, is disclosed by the presence of a slight potential difference across the conductor sample in a direction perpendicular to both the magnetic field and the velocity of the charges. The voltage is known as the Hall voltage, and the effect itself is called the Hall effect.

Figure illustrates the direction of the Hall voltage for both positive and negative charges in motion. In Figure 8.1a, \( \mathbf{v} \) is in the \(-ax\) direction, \( \mathbf{v} \times \mathbf{B} \) is in the \(ay\) direction, and \( Q \) is positive, causing \( FQ \) to be in the \(ay\) direction; thus, the positive charges move to the right. In Figure b, \( \mathbf{v} \) is now in the \(+ax\) direction, \( \mathbf{B} \) is still in the \(az\) direction, \( \mathbf{v} \times \mathbf{B} \) is in the \(-ay\) direction, and \( Q \) is negative; thus, \( FQ \) is again in the \(ay\) direction. Hence, the negative charges end up at the right
edge. Equal currents provided by holes and electrons in semiconductors can therefore be differentiated by their Hall voltages. This is one method of determining whether a given semiconductor is \( n \)-type or \( p \)-type. Devices employ the Hall effect to measure the magnetic flux density and, in some applications where the current through the device can be made proportional to the magnetic field across it, to serve as electronic watt meters, squaring elements, and so forth.

We defined convection current density in terms of the velocity of the volume charge density,

\[
J = \rho v
\]

The differential element of charge in (4) may also be expressed in terms of volume charge density,

\[
dQ = \rho v \, dv
\]

Thus

\[
dF = \rho v \, dv \times B
\]

or

\[
dF = J \times B \, dv
\]

that \( J \, dv \) may be interpreted as a differential current element; that is,

\[
J \, dv = K \, dS = I \, dL
\]
and thus the Lorentz force equation may be applied to surface current density,
\[ dF = K \times B \, dS \]
or to a differential current filament,
\[ dF = I \, dL \times B \]

2. Explain the Magnetic dipole and dipole moment?

Two point charges, one with charge \( +q \) and the other one with charge \( -q \) separated by a distance \( d \), constitute an electric dipole (a special case of an electric multipole). For this case, the electric dipole moment has a magnitude

and is directed from the negative charge to the positive one. Some authors may split \( d \) in half and use \( s = d/2 \) since this quantity is the distance between either charge and the center of the dipole, leading to a factor of two in the definition.

A stronger mathematical definition is to use vector algebra, since a quantity with magnitude and direction, like the dipole moment of two point charges, can be expressed in vector form

where \( d \) is the displacement vector pointing from the negative charge to the positive charge. The electric dipole moment vector \( p \) also points from the negative charge to the positive charge.

An idealization of this two-charge system is the electrical point dipole consisting of two (infinite) charges only infinitesimally separated, but with a finite \( p \).

Potential due to an electric dipole

We already know that electric dipole is an arrangement which consists of two equal and opposite charges \( +q \) and \( -q \) separated by a small distance \( 2a \).

Electric dipole moment is represented by a vector \( p \) of magnitude \( 2qa \) and this vector points in direction from \( -q \) to \( +q \).

To find electric potential due to a dipole consider charge \( -q \) is placed at point \( P \) and charge \( +q \) is placed at point \( Q \) as shown below in the figure.
Since electric potential obeys superposition principle so potential due to electric dipole as a whole would be sum of potential due to both the charges \( +q \) and \( -q \). Thus

\[
\nu = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{\ell_1} - \frac{q}{\ell_2} \right)
\]

The electric dipole moment for a pair of opposite charges of magnitude \( q \) is defined as the magnitude of the charge times the distance between them and the defined direction is toward the positive charge. ... Applications involve the electric field of adipole and the energy of a dipole when placed in an electric field.

Dipole Moment

The electric dipole moment for a pair of opposite charges of magnitude \( q \) is defined as the magnitude of the charge times the distance between them and the defined direction is toward the positive charge. It is a useful concept in atoms and molecules where the effects of charge separation are measurable, but the distances between the charges are too small to be easily
measurable. It is also a useful concept in dielectrics and other applications in solid and liquid material.

\[ \vec{p} = q \vec{d} \]

Applications involve the electric field of a dipole and the energy of a dipole when placed in an electric field.

\[ E_{dipole} = \frac{1}{4\pi\varepsilon_0} \frac{P}{z^3} \]

3. Derive the differential current loop as a magnetic dipole

Consider the application of a vertically upward force at the end of a horizontal crank handle on an elderly automobile. This cannot be the only applied force, for if it were, the entire handle would be accelerated in an upward direction. A second force, equal in magnitude to that exerted at the end of the handle, is applied in a downward direction by the bearing surface at the axis of rotation. For a 40-N force on a crank handle 0.3 m in length, the torque is 12 N·m. This figure is obtained regardless of whether the origin is considered to be on the axis of rotation (leading to 12 N·m plus 0 N·m), at the midpoint of the handle (leading to 6 N·m plus 6 N·m), or at some point not even on the handle or an extension of the handle.

We may therefore choose the most convenient origin, and this is usually on the axis of rotation and in the plane containing the applied forces if the several forces are coplanar.

With this introduction to the concept of torque, let us now consider the torque on a differential current loop in a magnetic field \( \mathbf{B} \). The loop lies in the \( x y \) plane; the sides of the loop are
parallel to the $x$ and $y$ axes and are of length $dx$ and $dy$. The value of the magnetic field at the center of the loop is taken as $B_0$.

A differential current loop in a magnetic field $B$. The torque on the loop is

$$dT = I (dx \, dy \, az) \times B_0 = I \, dS \times B,$$

where $dS$ is the vector area of the differential current loop and the subscript on $B_0$ has been dropped.

We now define the product of the loop current and the vector area of the loop as the differential 

*magnetic dipole moment* $d\mathbf{m}$, with units of A$ \cdot$ m$^2$. Thus

$$d\mathbf{m} = I \, dS$$

4. *Torque on a current loop placed in a magnetic field*  

Scalar Magnetic

We have already obtained general expressions for the forces exerted on current systems. One special case is easily disposed of, for if we take our relationship for therefore on a filamentary closed circuit, as given by Eq.

$$\mathbf{F} = -I \oint \mathbf{B} \times d\mathbf{L}$$
and assume a uniform magnetic flux density, then $\mathbf{B}$ may be removed from the integral

$$\mathbf{F} = -\mathbf{lB} \times \oint d\mathbf{l}$$

However, we discovered during our investigation of closed line integrals in an electrostatic potential field that

$d\mathbf{l} = 0$, and therefore the force on a closed filamentary circuit in a uniform magnetic field is zero.

If the field is not uniform, the total force need not be zero.

This result for uniform fields does not have to be restricted to filamentary circuits only. The circuit may contain surface currents or volume current density as well. If the total current is divided into filaments, the force on each one is zero, as we have shown, and the total force is again zero. Therefore, any real closed circuit carrying direct currents experiences a total vector force of zero in a uniform magnetic field.

Although the force is zero, the torque is generally not equal to zero. In defining the torque, or moment, of a force, it is necessary to consider both an origin at or about which the torque is to be calculated, and the point at which the force is applied. In Figure 8.5a, we apply a force $\mathbf{F}$ at point $P$, and we establish an origin at $O$ with a rigid lever arm $\mathbf{R}$ extending from $O$ to $P$. The torque about point $O$ is a vector whose magnitude is the product of the magnitudes of $\mathbf{R}$, of $\mathbf{F}$, and of the sine of the angle between these two vectors. The direction of the vector torque $\mathbf{T}$ is normal to both the force $\mathbf{F}$ and the lever arm $\mathbf{R}$ and is in the direction of progress of a right-handed screw as the lever arm is rotated into the force vector through the smaller angle. The torque is expressible as a cross product,
5. Explain the vector magnetic potential and its properties vector magnetic potential due to simple configurations

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity \( \vec{H} = -\nabla V_m \) to a scalar magnetic potential and write: From Ampere's law, we know that

\[
\nabla \times \vec{H} = \vec{J} \quad \text{Therefore,} \quad \nabla \times (-\nabla V_m) = \vec{J}
\]

But using vector identity, \( \nabla \times (\nabla V) = 0 \) we find that \( \vec{H} = -\nabla V_m \) is valid only where \( \vec{J} = 0 \). Thus the scalar magnetic potential is defined only in the region where \( \vec{J} = 0 \). Moreover, \( V_m \) in general is not a single valued function of position. This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig

In the region \( a < \rho < b \), \( \vec{J} = 0 \) and

\[
\vec{H} = \frac{I}{2\pi \rho} \hat{\phi}
\]

Cross Section of a Coaxial Line

If \( V_m \) is the magnetic potential then,
\[-\nabla V_m = -\frac{1}{\rho} \frac{\partial V_m}{\partial \phi} \]
\[= \frac{I}{2\pi \rho} \]

\[\therefore V_m = -\frac{I}{2\pi} \phi + c \quad \quad \quad V_m = -\frac{I}{2\pi} \phi \]

If we set \( V_m = 0 \) at \( \phi = 0 \) then \( c = 0 \) and

We observe that as we make a complete lap around the current carrying conductor, we
reach \( \phi_0 \) again but \( V_m \) this time becomes
\[V_m = -\frac{I}{2\pi} (\phi_0 + 2\pi)\]

We observe that value of \( V_m \) keeps changing as we complete additional laps to pass through
the same point. We introduced \( V_m \) analogous to electrostatic potential \( V \). But for static electric
fields, \( \nabla \times \vec{E} = 0 \) and \( \oint \vec{E} \cdot d\vec{l} = 0 \), whereas for steady magnetic
field \( \nabla \times \vec{H} = 0 \) wherever \( \vec{J} = 0 \) but \( \oint \vec{H} \cdot d\vec{l} = I \) even if \( \vec{J} = 0 \) along the path of integration.

\[\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi} \phi_0 \]

We now introduce the vector magnetic potential which can be
used in regions where current density may be zero or nonzero and the same can be easily
extended to time varying cases. The use of vector magnetic potential provides elegant ways of
solving EM field problems. Since \( \nabla \cdot \vec{B} = 0 \) and we have the vector identity that for any
vector \( \vec{A} \), \( \nabla \cdot (\nabla \times \vec{A}) = 0 \), we can write \( \vec{B} = \nabla \times \vec{A} \).

Here, the vector field \( \vec{A} \) is called the vector magnetic potential. Its SI unit is Wb/m. Thus if
we can find \( \vec{A} \) of a given current distribution, \( \vec{B} \) can be found from \( \vec{A} \) through a curl operation.

We have introduced the vector function \( \vec{A} \) and related its curl to \( \vec{B} \). A vector function is
defined fully in terms of its curl as well as divergence. The choice of \( \nabla \vec{A} \) is made as follows.
\[
\nabla \times \nabla \times \vec{A} = \mu \nabla \times \vec{H} = \mu \vec{J}.
\]

**Objective questions**

The tangential component of the magnetic field intensity is continuous at the boundary of separation of two media. State True/False.

a) True

b) False

**Answer: a**

3. In air, the tangential component of flux density is continuous at the boundary. State True/False.

a) True

b) False

**Answer: a**

4. The inductance is proportional to the ratio of flux to current. State True/False.

a.) True

b) False

**Answer: a**

5. A coil is said to be loosely coupled with which of the following conditions?

a) \( K > 1 \)

b) \( K < 1 \)

c) \( K > 0.5 \)
d) K<0.5
Answer: d

6. The equivalent inductances of two coils 2H and 5H in series aiding flux with mutual inductance of 3H is
a) 10
b) 30
c) 1
 d) 13
Answer: d

7. The expression for the inductance in terms of turns, flux and current is given by
a) \( L = N \frac{\Delta \phi}{\Delta i} \)
b) \( L = - N \frac{\Delta \phi}{\Delta i} \)
c) \( L = N i \phi \)
d) \( L = N \phi / i \)
Answer: a

---

Fill In The Blanks

1. ___________is always zero for static fields Which of the following identities is
always zero for static fields

2. Maxwell first law value for the electric field intensity is given by $A \sin \omega t \, az$ is

3. Maxwell second equation is based on

4. continuity equation is

5. The total current density is given as $0.5i + j - 1.5k$ units. Find the curl of the magnetic field intensity is

6. conduction and displacement current densities coexist in

7. The displacement current density when the electric flux density is $20\sin 0.5t$ is

8. Displacement current density in air at a frequency of 18GHz in frequency domain. Take electric field $E$ as 4 units is

9. conduction and displacement currents become equal with unity conductivity in a material of permittivity 2 is

10. The ratio of conduction to displacement current density is referred to as

ANSWERS

1. Curl(Grad V)  2. 0  3. Amperes law  4. $J = \sigma \, E$  5. $0.5i + j - 1.5k$

6. Conductors placed in any dielectric medium  7. $10\cos 0.5t$  8. 4  9.9  10. Loss tangent

UNIT 5

TIME VARYING FIELDS

2-Marks Question and answers
1. **Give the characteristic impedance of free space?**
   
   Ans: 377ohms.

2. **Define propagation constant.**
   
   Ans: Propagation constant is a complex number $\gamma = \alpha + j\beta$ where $\gamma$ is propagation constant.

3. **Give the difficulties in FDM?**
   
   FDM is difficult to apply for problems involving irregular boundaries and non-homogeneous material properties.

4. **State Maxwell's fourth equation?**
   
   The net magnetic flux emerging through any closed surface is zero.

   Three marks

3-Marks Question and answers

1. **State the principle of superposition of fields?**
   
   The total electric field at a point is the algebraic sum of the individual electric field at that point.

2. **Define loss tangent?**
   
   Loss tangent is the ratio of the magnitude of conduction current density to displacement current density of the medium.

3. **Define a wave?**
   
   If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location then the group of phenomena constitutes a wave.

4. **Write properties of uniform plane wave.**
   
   i) At every point in space, the electric field $E$ and magnetic field $H$ are perpendicular to each other.

   ii) The fields vary harmonically with time and at the same frequency everywhere in space.
5. **Explain the steps in finite element method.**

i) Discrimination of the solution region into elements.

ii) Generation of equations for fields at each element.

iii) Assembly of all elements.

iv) Solution of the resulting system.

5-Marks Question and answers

1. **State and explain the Faraday’s laws of electromagnetic induction?**

Faraday’s Law of electromagnetic Induction

Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed. In terms of fields, we can say that a time varying magnetic field produces an electromotive force (emf) which causes a current in a closed circuit. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday’s law. Mathematically, the induced emf can be written as

\[
\text{Emf} = -\frac{d\phi}{dt} \quad \text{Volts}
\]

Where \( \phi \) is the flux linkage over the closed path.

A non zero \( \frac{d\phi}{dt} \) may result due to any of the following:

(a) Time changing flux linkage a stationary closed path.

(b) Relative motion between a steady flux a closed path.

(c) A combination of the above two cases.

The negative sign in equation 3 was introduced by Lenz in or
Consider to comply with the polarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of \( N \) tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

\[
Emf = -N \frac{d\phi}{dt} \quad \text{Volts}
\]

By defining the total flux linkage as

\[ \lambda = N\phi \]

The emf can be written as

\[
Emf = -\frac{d\lambda}{dt}
\]

Continuing with equation (5.3), over a closed contour ‘C’ we can write

\[
Emf = \oint_C \vec{E} \cdot d\vec{l}
\]

Where \( \vec{E} \) is the induced electric field on the conductor to sustain the current.

2. Explain the Maxwell’s equations in time varying fields?

Maxwell’s fourth equation, Curl (\( E \))=–B/\( t \) – Statically and Dynamically induced EMFs:

Further, total flux enclosed by the contour ‘C’ is given by
\[ \phi = \oint_S \vec{B} \cdot d\vec{s} \]

Where \( S \) is the surface for which 'C' is the contour.

From and using in we can write

\[ \oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \oint_S \vec{B} \cdot d\vec{s} \]

By applying stokes theorem

\[ \int_S \nabla \times \vec{E} \cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \]

Therefore, we can write

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

which is the Faraday's law in the point form

\[ \frac{d\phi}{dt} \]

We have said that non zero \( \frac{d\phi}{dt} \) can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf.

3. Derive the Maxwell's fourth equation?

Maxwell’s fourth equation, Curl (E)=−B/t – Statically and Dynamically induced EMFs:

Further, total flux enclosed by the contour 'C' is given by

\[ \phi = \oint_S \vec{B} \cdot d\vec{s} \]

Where \( S \) is the surface for which 'C' is the contour.
From and using in we can write
\[ \oint_c \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \Phi \oint_s \mathbf{B} \cdot d\mathbf{s} \]

By applying Stokes theorem
\[ \int_s \nabla \times \mathbf{E} \cdot d\mathbf{s} = \oint_s \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \]

Therefore, we can write
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

which is the Faraday's law in the point form
\[ \frac{d\Phi}{dt} \]

We have said that non zero \( \frac{d\Phi}{dt} \) can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf.

4. **Express the time varying equations?**

In our study of static fields so far, we have observed that static electric fields are produced by electric charges, static magnetic fields are produced by charges in motion or by steady current. Further, static electric field is a conservative field and has no curl, the static magnetic field is continuous and its divergence is zero. The fundamental relationships for static electric fields among the field quantities can be summarized as:
\[ \nabla \times \mathbf{E} = 0 \]
\[ \nabla \cdot \mathbf{D} = \rho_v \]

For a linear and isotropic medium,
\[ \mathbf{D} = \varepsilon \mathbf{E} \]
Similarly for the magneto static case

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{H} = \vec{J} \]

\[ \vec{B} = \mu \vec{H} \]

It can be seen that for static case, the electric field vectors \( \vec{E} \) and \( \vec{D} \) and magnetic field vectors \( \vec{B} \) and \( \vec{H} \) form separate pairs.

In this chapter we will consider the time varying scenario. In the time varying case we will observe that a changing magnetic field will produce a changing electric field and vice versa.

We begin our discussion with Faraday's Law of electromagnetic induction and then present the Maxwell's equations which form the foundation for the electromagnetic theory.

**Objective Type Questions**

1. For time varying currents fields are
   a) Electrostatic
   b) Magneto static
   c) Electromagnetic
   d) Electrical

   **Answer:** c

2. According to faradays law, EMF stands for
   a) Electromagnetic field
b) Electromagnetic

c) Electromagnetic friction

d) Electromotive force

**Answer:** d.

3. The induced voltage opposes flux producing it. State true or false

a) True

b) False

**Answer:** a

4. Find the displacement current when the flux density is given by \( t^3 \) at 2 seconds.

a) 3

b) 6

c) 12

d) 27

**Answer:** c.

5. Calculate the emf when a coil of 100 turns is subjected to a flux rate of 0.3 tesla/sec.

a) 3

b) 30

c) -30

d) -300

**Answer:** c

**Fill In The Blanks**

1. The dipole formation in a magnet is due to----------?

2. Calculate the emf in a material with flux linkage of \( 3.5t^2 \) at 2 seconds---------?

3. The line integral of the electric field intensity is----------------?

4. For static fields, the curl of E will be-----------?

5. The line integral of which parameter is zero for static fields-----------?

6. The charge density of a field with a position vector as electric flux density is given by---------- ---------?
7. Which of the following relations is correct? 

8. The charge build up in a capacitor is due to? 

9. The propagation of the electromagnetic waves can be illustrated by? 

10. Which one of the following laws will not contribute to the Maxwell’s equations? 

ANSERS

1. Interaction between the north and south poles together 
2. -14 

17. BEYOND SYLLABUS TOPICS WITH MATERIAL

TIME VARYING FIELDS APPLICATIONS

Potential Formulation of Maxwel’s Equations

We have arrived at the set of Maxwell’s equations, which are,

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \leftrightarrow \quad \nabla \cdot \vec{B} = \rho_{free} \]

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \nabla \times \vec{H} = \vec{j}_{free} + \frac{\partial \vec{D}}{\partial t} \quad \leftrightarrow \quad \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \]

long with the constitutive relations,
In this lecture will attempt a formulation of the problem in terms of potentials. It may be observed that Maxwell’s equations are four equations for six quantities, viz., the components of \( \mathbf{E} \) and \( \mathbf{B} \).

Let us recall that in electrostatics, we had defined a scalar potential by \( \mathbf{E} = -\nabla \varphi \) and in magnetostatics we introduced a vector potential \( \mathbf{A} \).

Faraday’s law gives us

\[
\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} = -\frac{\partial}{\partial t} (\nabla \times \mathbf{A})
\]

which gives us

\[
\nabla \times \left( \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0
\]

Notice that since with the introduction of Faraday’s law, the electric field did not remain conservative, we have not replaced the electric field with the gradient of a scalar function. We have still retained a vector potential because the magnetic field was a general vector field and it can always be expressed as a curl of yet another vector. However, the vector potential would not be the same as it was in the magnetostatic case.

Since the curl of \( \mathbf{A} \) is zero, this combination is conservative and being an irrotational field can be expressed as the gradient of a scalar function \( \varphi \). We define,

\[
\mathbf{E} = -\nabla \varphi - \frac{\partial \mathbf{A}}{\partial t}
\]

Let us see what this does to the Maxwell’s equations.

In vacuum, we have,

\[
\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}
\]
In terms of \( V \) and \( \vec{A} \), we have, on taking the divergence of both sides of the preceding equation,

\[
\nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \vec{A}) = -\frac{\rho}{\epsilon_0}
\]

which contains both the electrostatics Gauss’s law and the Faraday’s law.

Let us consider the remaining pair of Maxwell’s equations. The divergence of magnetic field continues to be zero even after introduction of time varying electric field. However, Ampere’s law

\[
\nabla \times \vec{B} = \mu_0 \vec{I} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}
\]

can be restated as follows:

\[
\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{I} + \frac{1}{c^2} \frac{\partial}{\partial t} \left( -\nabla V - \frac{\partial \vec{A}}{\partial t} \right)
\]

where we have used, \( \mu_0 \epsilon_0 = \frac{1}{c^2} \), \( c \) being the speed of light in vacuum, a universal constant.

We can rewrite this equation, by rearranging terms, as

\[
\left( \nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \right) - \nabla \left( \nabla \cdot \vec{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} \right) = -\mu_0 \vec{I}
\]

Equations (1) and (2) are two equations in four quantities (three components of \( A \) and one component of \( V \)) which are equivalent to the four Maxwell’s equations that we had obtained. These equations are not decoupled.

We will now use a choice of gauge that we have for the potential. We know that the scalar potential \( V \) is undetermined up to a constant while we can add gradient of a scalar function to the vector potential.

We will use,
subject to Lorentz Gauge condition,
\[
\hat{A} \rightarrow \hat{A}' = \hat{A} + \nabla \psi \\
V \rightarrow V' = V - \frac{\partial \psi}{\partial t}
\]
\[
\nabla \cdot \hat{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} = 0
\]

the equations (1) and (2) get decoupled and give a pair of inhomogeneous wave equations

\[
\nabla^2 V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0}
\]
\[
\nabla^2 \hat{A} - \frac{1}{c^2} \frac{\partial^2 \hat{A}}{\partial t^2} = -\mu_0 \hat{J}
\]
solutions of which can always be found. However, can we ensure that Lorentz gauge can always be satisfied? The answer is yes. Suppose we have a pair of \( V \) and \( \hat{A} \) for which Lorentz gauge is not satisfied and we have,

\[
\nabla \cdot \hat{A} + \frac{1}{c^2} \frac{\partial V}{\partial t} = f(\vec{r}, t) \neq 0
\]

where \( f(\vec{r}, t) \) is a scalar function of space and time. Let us define

\[
\hat{A}' = \hat{A} + \nabla \psi \\
V' = V - \frac{\partial \psi}{\partial t}
\]

In terms of these, we have,

\[
\nabla \cdot (\hat{A}' - \nabla \psi) + \frac{1}{c^2} \frac{\partial V'}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = f(\vec{r}, t)
\]

We can get Lorentz gauge to be satisfied for \( \hat{A}' \) and \( V' \) if we have,

\[
\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = -f(\vec{r}, t)
\]

which is equation for which a solution can always be found. Thus Lorentz gauge condition can always be satisfied.

energy Density of electromagnetic field
Let us calculate the energy density of electromagnetic field and see how the energy contained in such field can change in a closed volume.

we have seen that the energy of a collection of charges can be written as follows:

\[
W_{el} = \frac{1}{2} \int \rho(\vec{r}) \varphi(\vec{r}) d^3r \\
= \frac{1}{2} \int (\nabla \cdot \vec{D}) \varphi(\vec{r}) d^3r \\
= -\frac{1}{2} \int (\vec{D} \cdot \nabla \varphi) d^3r \\
= \frac{1}{2} \int \vec{E} \cdot \vec{D} d^3r
\]

where in the penultimate step, we have used the relation

\[
\varphi \nabla \cdot \vec{D} = \nabla \cdot (\varphi \vec{D}) - \vec{D} \cdot \nabla \varphi
\]

and converted the integral of the first term on the right to a surface integral by the divergence theorem and discarded the surface integral by taking the surface to infinity so that the remaining integral is all over space. The electric energy density can be thus written as

\[
u_E = \frac{\vec{E} \cdot \vec{D}}{2} \rightarrow \frac{\varepsilon E^2}{2}
\]

the last relation being true for a linear electric medium.

Likewise, the magnetic energy can be written as

\[
W_{mag} = \frac{1}{2} \int \vec{A} \cdot \vec{J} d^3r \\
= \frac{1}{2} \int \vec{A} \cdot (\nabla \times \vec{H}) d^3r \\
= \frac{1}{2} \int \vec{H} \cdot (\nabla \times \vec{A}) d^3r \\
= \frac{1}{2} \int \vec{E} \cdot \vec{B} d^3r
\]
The magnetic energy density is given by

\[ u = \frac{\mathbf{E} \cdot \mathbf{D}}{2} + \frac{\mathbf{H} \cdot \mathbf{B}}{2} \]

The total energy density in electromagnetic field is thus given by

Poynting Theorem

Consider the case of an electromagnetic field confined to a given volume. How does the energy contained in the field change? There are two processes by which it can happen. The first is by the mechanical work done by the electromagnetic field on the currents, which would appear as Joule heat and the second process is by radioactive flow of energy across the surface of the volume.

The mechanical power is given by

\[ P_{\text{mech}} = \int \mathbf{E} \cdot \mathbf{v} \, d^3r \]

\[ = \int \rho(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \, d^3r \]

where the work done by the magnetic field is zero. This amount of energy appears as Joule heat.

The rate of change of energy in the volume can be calculated as follows. For a linear electric and linear magnetic medium,

\[ \frac{dW}{dt} = \int \frac{\partial u}{\partial t} \, d^3r \]

Define “Poynting Vector”

\[ \frac{1}{2} \int \frac{\partial}{\partial t} \left[ \frac{D^2}{2\varepsilon} + \frac{B^2}{2\mu} \right] \, d^3r \]

\[ = \int \left[ \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right] \, d^3r \]

\[ = \int \left[ \mathbf{E} \cdot (\nabla \times \mathbf{H} - \mathbf{j}_{\text{free}}) - \mathbf{H} \cdot (\nabla \times \mathbf{E}) \right] \, d^3r \]

We have,

\[ = \int \left( \mathbf{E} \cdot (\nabla \times \mathbf{H}) - \mathbf{H} \cdot (\nabla \times \mathbf{E}) \right) \, d^3r - \frac{1}{2} \int \mathbf{E} \cdot \mathbf{j} \, d^3r \]

\[ = \int \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, d^3r - \frac{1}{2} \int \mathbf{E} \cdot \mathbf{j} \, d^3r \]

\[ \frac{dW}{dt} \equiv \int \frac{du}{dt} \, d^3r = -\frac{1}{2} \int \nabla \cdot \mathbf{S} \, d^3r - \int \mathbf{E} \cdot \mathbf{j} \, d^3r \]
As the relation is valid for arbitrary volume, we have
\[
\frac{du}{dt} + \nabla \cdot \vec{\mathcal{E}} = -\vec{E} \cdot \vec{j}
\]

This equation represents the energy conservation equation of electrodynamics.

Converting the term representing volume integral of the Pointing vector to a surface integral, we get where we have denoted the surface element by da instead of the usual dS so that it does not cause confusion with our notation for the Pointing vector.