3) Course Objectives, Course Outcomes and Topic Outcomes

a) Course Objectives

1. Define the basics of Signals and Systems required for all Electrical Engineering related courses.
2. Discuss concepts of Signals and Systems and its analysis using different transform techniques.
3. Describe the concept of random process which is essential for random signals and systems encountered in Communications and Signal Processing areas.

b) Course Outcomes

At the end of the course student will be able to

1. Identify the importance of orthogonal concept and standard functions.
2. Develop any arbitrary analog or Digital time domain signal in frequency domain.
3. Recognize the characteristics of linear time invariant systems.
4. Differentiate the Laplace transform and Z-transform.
5. Illustrate the sampling concept and Density spectrum.

C) Topic outcomes

S.No	Topic	**Topic outcome** At the end of the topic the student will be able to
	UNIT-I **Signal Analysis**	
1	Fundamentals of signals and systems	Define the signal and system
2	Classification of signals and systems	Classify the different signals and systems
3	Operations on signals	Determine the operations on signals
4	**Signal Analysis:** Analogy between Vectors and Signals.	Define signal, system and orthogonal signal space
5	Orthogonal Signal Space.	Define orthogonal signal space
6	Signal approximation using Orthogonal functions, Mean Square Error.	Approximate the signal using orthogonal function.Define mean square error.
7	Closed or complete set of Orthogonal functions.	Define the complete set of orthogonal functions.
8	Orthogonality in Complex functions.	Define Orthogonality in complex functions
9	Exponential and Sinusoidal signals Concepts of Impulse function,	Define the basic signals in graphical as well as functional representation
10	Concepts of Impulse function, Unit Step function, Signum function.	Define the basic signals in graphical as well as functional representation
11	Classifications of Signals.	Define the different classifications of signals. Compare the energy and power signal.
12	Classifications of Systems.	Define linear system. Define linear time invariant (LTI) t system.
	UNIT-II **Fourier series &Transforms**	
13	**Introduction** **Fourier series:** Representation of Fourier series Continuous time periodic signals.	Define any periodic signal in terms of Fourier series
14	Properties of Fourier Series.	Define the properties of Fourier series
15	Dirichlet's conditions,	Define the Dirichlet's condition with examples
16	Trigonometric Fourier Series Exponential Fourier Series.	Relate any periodic signal in terms of Trigonometric, exponential forms of Fourier Series
17	Complex Fourier Series.	Derive The complex Fourier series

18	**Fourier Transforms:** Deriving Fourier Transform from Fourier series.	Derive Fourier transform from Fourier series
19	Fourier Transform of arbitrary signal.	Determine the Fourier transform of arbitrary signal
20	Fourier Transform of standard signals.	Determine the Fourier transform of standard signals
21	Fourier Transform of Periodic Signals.	Derive the Fourier Transform of Periodic Signals
22	Properties of Fourier Transform.	Define the Properties of Fourier Transform.
23	Fourier Transform involving impulse and Signum function	Derive the Fourier Transform involving impulse and Signum function
24	Problems on Fourier transform and inverse Fourier transform.	Determine Fourier transform the different signals like triangular signal, sinwt, coswt etc
25	Introduction to Hilbert Transform	Define the Hilbert Transform
	UNIT-III **Signal Transmission through Linear Systems**	
26	Linear System	Define the Linear System
27	Impulse response of a Linear System	Determine the Impulse response of a Linear System
28	Linear Time Invariant(LTI) System	Define the Linear Time Invariant(LTI) System
29	Linear Time Variant (LTV) System	Define the Linear Time Variant (LTV) System
30	Transfer function of a LTI System	Determine the Transfer function of a LTI System
31	Filter characteristic of Linear System	Describe the Filter characteristic of Linear System
32	Distortion less transmission through a system, Signal bandwidth	Define the Distortion less transmission through a system, Signal bandwidth
33	System Bandwidth, Ideal LPF, HPF, and BPF characteristics	Explain the System Bandwidth, Ideal LPF, HPF, and BPF characteristics
34	Causality and Paley-Wiener criterion for physical realization	State and prove the Causality and Paley-Wiener criterion for physical realization
35	Relationship between Bandwidth and rise time	Explain the Relationship between Bandwidth and rise time
36	Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain	Define the Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain
37	Graphical representation of Convolution.	Define the Graphical representation of Convolution.
	UNIT-IV **Laplace Transforms & Z-Transforms**	
38	**Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform	**Define the** Laplace Transforms (L.T), Inverse Laplace Transform

39	Concept of Region of Convergence (ROC) for Laplace Transforms	Define Concept of Region of Convergence (ROC) for Laplace Transforms
40	Properties of **Laplace Transform**	State and prove the Properties of **Laplace Transform**
41	Relation between L.T and F.T of a signal	Determine the Relation between L.T and F.T of a signal
42	Laplace Transform of certain signals using waveform synthesis	Determine the Laplace Transform of certain signals using waveform synthesis
43	**Z–Transforms:** Concept of Z- Transform of a Discrete Sequence	**Define the** Concept of Z- Transform of a Discrete Sequence
44	Distinction between Laplace, Fourier and Z Transforms	Distinguish between Laplace, Fourier and Z Transforms
45	Region of Convergence in Z-Transform	Define Region of Convergence in Z-Transform
46	Constraints on ROC for various classes of signals	Determine the Constraints on ROC for various classes of signals
47	Inverse Z-transform, Properties of Z-transforms	State and prove the Inverse Z-transform, Properties of Z-transforms
	UNIT-V **Sampling theorem & Correlation**	
48	Graphical and analytical proof for Band Limited Signals	State and prove the Graphical and analytical proof for Band Limited Signals
49	Impulse Sampling, Natural and Flat top Sampling	Define the Impulse Sampling, Natural and Flat top Sampling
50	Reconstruction of signal from its samples	Explain the Reconstruction of signal from its samples
51	Effect of under sampling – Aliasing	Explain Effect of under sampling – Aliasing
52	Introduction to Band Pass Sampling	Define the Band Pass Sampling
53	Cross Correlation and Auto Correlation of Functions	Determine the Cross Correlation and Auto Correlation of Functions
54	Properties of Correlation Functions, Energy Density Spectrum	State and prove the Properties of Correlation Functions, Energy Density Spectrum
55	Parsevals Theorem, Power Density Spectrum	State and prove Parsevals Theorem, Power Density Spectrum
56	Relation between Autocorrelation Function and Energy/Power Spectral Density Function	Explain the Relation between Autocorrelation Function and Energy/Power Spectral Density Function
57	Relation between Convolution and Correlation	State and prove the Relation between Convolution and Correlation
58	Detection of Periodic Signals in the presence of Noise by Correlation	Explain Detection of Periodic Signals in the presence of Noise by Correlation
59	Extraction of Signal from Noise by Filtering	Determine the Extraction of Signal from Noise by Filtering
60	Filter design	Design the different types of filters

61	AWG noise characteristics	Explain the AWG noise characteristics

4) COURSE PRE–REQUISITES

1. Basics of signals and systems
2. Classifications of signals and system
3. Operations on signals
4. Introduction to vectors

5) Course Information Sheet

5.a). COURSE DESCRIPTION:

PROGRAMME: B. Tech. (Electronics and Communications Engineering.)	DEGREE: B.TECH
COURSE: **SIGNALS AND SYSTEMS**	YEAR: II SEM: I CREDITS: 4
COURSE CODE: EC304PC REGULATION: R18	COURSE TYPE: CORE
COURSE AREA/DOMAIN: Basics in signal processing	CONTACT HOURS: 3+1 (L+T)) hours/Week.
CORRESPONDING LAB COURSE CODE (IF ANY):EC307PC	LAB COURSE NAME: BS LAB

5.b). SYLLABUS:

Unit	Details	Hours
I	**Signal Analysis:** Analogy between Vectors and Signals, Orthogonal Signal Space, Signal approximation using Orthogonal functions, Mean Square Error, Closed or complete set of Orthogonal functions, Orthogonality in Complex functions, Classification of Signals and systems, Exponential and Sinusoidal signals, Concepts of Impulse function, Unit Step function, Signum function.	10
II	**Fourier series:** Representation of Fourier series, Continuous time periodic signals, Properties of Fourier Series, Dirichlet's conditions, Trigonometric Fourier Series and Exponential Fourier Series, Complex Fourier spectrum. **Fourier Transforms:** Deriving Fourier Transform from Fourier series, Fourier Transform of arbitrary signal, Fourier Transform of standard signals, Fourier Transform of Periodic Signals, Properties of Fourier Transform, Fourier Transforms involving Impulse function and Signum function, Introduction to Hilbert Transform.	13
III	**Signal Transmission through Linear Systems:** Linear System, Impulse response, Response of a Linear System, Linear Time Invariant(LTI) System, Linear Time Variant (LTV) System, Transfer function of a LTI System, Filter characteristic of Linear System, Distortion less transmission through a system, Signal bandwidth, System Bandwidth, Ideal LPF, HPF, and BPF characteristics, Causality and Paley-Wiener criterion for physical realization, Relationship between Bandwidth and rise time, Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain, Graphical representation of Convolution.	12
IV	**Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform, Concept of Region of Convergence (ROC) for Laplace Transforms, Properties of L.T, Relation between L.T and F.T of a signal, Laplace Transform of certain signals using waveform synthesis. **Z–Transforms:** Concept of Z- Transform of a Discrete Sequence, Distinction between Laplace, Fourier and Z Transforms, Region of Convergence in Z-Transform, Constraints on ROC for various classes of signals, Inverse Z-transform, Properties of Z-transforms.	10

V	**Sampling theorem**: Graphical and analytical proof for Band Limited Signals, Impulse Sampling, Natural and Flat top Sampling, Reconstruction of signal from its samples, Effect of under sampling – Aliasing, Introduction to Band Pass Sampling. **Correlation:** Cross Correlation and Auto Correlation of Functions, Properties of Correlation Functions, Energy Density Spectrum, Parsevals Theorem, Power Density Spectrum, Relation between Autocorrelation Function and Energy/Power Spectral Density Function, Relation between Convolution and Correlation, Detection of Periodic Signals in the presence of Noise by Correlation, Extraction of Signal from Noise by Filtering.	12
	Contact classes for syllabus coverage	**57**
	Lectures beyond syllabus	**02**
	Tutorial classes	**09**
	Classes for gaps& Add-on classes	**02**
	Total No. of classes	**70**

5.c). GAPS IN THE SYLLABUS - TO MEET INDUSTRY/PROFESSION REQUIREMENTS:

S.NO.	DESCRIPTION	No. Of Classes
1	Fundamentals of signals and systems	1
2	Operation on signals	1

5. d). TOPICS BEYOND SYLLABUS / ADVANCED TOPICS:

S.NO.	DESCRIPTION	No. Of Classes
1	Filter design	1
2	AWG noise characteristics	1

5. e). WEB SOURCE REFERENCES:

Sl. No.	Name of book/ website
a.	nptel.ac.in/courses/112104121/1
b.	http://nptel.ac.in/courses/117101055/14
c.	http://nptel.ac.in/courses/117101055/36

5. f). DELIVERY / INSTRUCTIONAL METHODOLOGIES:

☑ CHALK & TALK	☑ STUD. ASSIGNMENT	☑ WEB RESOURCES
☑ LCD/SMART BOARDS	☑ STUD. SEMINARS	☐ ADD-ON COURSES

5.g). ASSESSMENT METHODOLOGIES - DIRECT

☑ ASSIGNMENTS	☑ STUD. SEMINARS	☑ TESTS/MODEL EXAMS	☑ UNIV. EXAMINATION
☑ STUD. LAB PRACTICES	☑ STUD. VIVA	☐ MINI/MAJOR PROJECTS	☐ CERTIFICATIONS
☐ ADD-ON COURSES	☐ OTHERS		

5.h). ASSESSMENT METHODOLOGIES - INDIRECT

☑ ASSESSMENT OF COURSE OUTCOMES (BY FEEDBACK, ONCE)	☑ STUDENT FEEDBACK ON FACULTY (TWICE)
☐ASSESSMENT OF MINI/MAJOR PROJECTS BY EXT. EXPERTS	☐ OTHERS

5.i). TEXT / REFERENCE BOOKS:

T/R	BOOK TITLE/AUTHORS/PUBLICATION
Text Book	Signals, Systems & Communications - B.P. Lathi , 2013, BSP.
Text Book	Signal and systems principles and applications, shaila dinakar Apten, Cambridez university press, 2016.
Text Book	Probability, Random Variables & Random Signal Principles - Peyton Z. Peebles, MCGRAW HILL EDUCATION, 4th Edition, 2001
Reference Book	Signals and Systems - A.V. Oppenheim, A.S. Willsky and S.H. Nawab, 2 Ed.,
Reference Book	Signals and Signals – Iyer and K. Satya Prasad, Cengage Learning
Reference Book	Signals and sytems- A Nagoor Kani, Mc Graw Hill

6. Micro Lesson Plan

Topic wise Coverage [Micro Lesson Plan]			
S.No.	**Topic**	**Scheduled date**	**Actual date**
	UNIT-I		
1	Fundamentals of signals and systems		
2	Classification of signals and systems		
3	Operations on signals		
4	**Signal Analysis:** Analogy between Vectors and Signals.		
5	Orthogonal Signal Space.		
6	Signal approximation using Orthogonal functions, Mean Square Error.		
7	Closed or complete set of Orthogonal functions.		
8	Orthogonality in Complex functions.		
9	Exponential and Sinusoidal signals Concepts of Impulse function,		
10	Concepts of Impulse function, Unit Step function, Signum function.		
11	Classifications of Signals.		
12	Classifications of Systems.		
13	Revision		
14	Slip test on Unit-I		
	UNIT-II		
15	**Introduction** **Fourier series:** Representation of Fourier series Continuous time periodic signals.		
16	Properties of Fourier Series.		
17	Dirichlet's conditions,		
18	Trigonometric Fourier Series Exponential Fourier Series.		
19	Complex Fourier Series.		

20	**Fourier Transforms:** Deriving Fourier Transform from Fourier series.		
21	Fourier Transform of arbitrary signal.		
22	Fourier Transform of standard signals.		
23	Fourier Transform of Periodic Signals.		
24	Properties of Fourier Transform.		
25	Fourier Transform involving impulse and Signum function		
26	Problems on Fourier transform and inverse Fourier transform.		
27	Introduction to Hilbert Transform		
28	Revision		
29	Slip Test		
	UNIT-III		
30	Linear System		
31	Impulse response of a Linear System		
32	Linear Time Invariant(LTI) System		
33	Linear Time Variant (LTV) System		
34	Transfer function of a LTI System		
35	Filter characteristic of Linear System		
36	Distortion less transmission through a system, Signal bandwidth		
37	System Bandwidth, Ideal LPF, HPF, and BPF characteristics		
38	Causality and Paley-Wiener criterion for physical realization		
39	Relationship between Bandwidth and rise time		
40	Convolution and Correlation of Signals, Concept of convolution in Time domain and Frequency domain		
41	Graphical representation of Convolution.		
42	Revision		

43	Slip Test		
	UNIT-IV		
44	**Laplace Transforms:** Laplace Transforms (L.T), Inverse Laplace Transform		
45	Concept of Region of Convergence (ROC) for Laplace Transforms		
46	Properties of **Laplace Transform**		
47	Relation between L.T and F.T of a signal		
48	Laplace Transform of certain signals using waveform synthesis		
49	**Z–Transforms:** Concept of Z- Transform of a Discrete Sequence		
50	Distinction between Laplace, Fourier and Z Transforms		
51	Region of Convergence in Z-Transform		
52	Constraints on ROC for various classes of signals		
53	Inverse Z-transform, Properties of Z-transforms		
54	Problems on Laplace Transform		
55	Problems on Z-Transform		
56	Slip Test		
	UNIT-V		
57	Graphical and analytical proof for Band Limited Signals		
58	Impulse Sampling, Natural and Flat top Sampling		
59	Reconstruction of signal from its samples		
60	Effect of under sampling – Aliasing		
61	Introduction to Band Pass Sampling		
62	Cross Correlation and Auto Correlation of Functions		
63	Properties of Correlation Functions, Energy Density Spectrum		
64	Parsevals Theorem, Power Density Spectrum		

65	Relation between Autocorrelation Function and Energy/Power Spectral Density Function		
66	Relation between Convolution and Correlation		
67	Detection of Periodic Signals in the presence of Noise by Correlation		
68	Extraction of Signal from Noise by Filtering		
69	Filter design & AWG noise characteristics		
70	Revision		

7. Teaching Schedule

Subject	SIGNALS AND STOCHASTIC PROCESS
	Text Books (to be purchased by the Students)
Book 1	Signals, Systems & Communications - B.P. Lathi , 2013, BSP.
Book 2	Signal and systems principles and applications, shaila dinakar Apten, Cambridezuniversity press, 2016.
Book 3	Probability, Random Variables & Random Signal Principles - Peyton Z. Peebles, MCGRAW HILL EDUCATION, 4th Edition, 2001
	Reference Books
Book 4	Signals and Systems - A.V. Oppenheim, A.S. Willsky and S.H. Nawab, 2 Ed.,
Book 5	Signals and Signals – Iyer and K. Satya Prasad, Cengage Learning
Book 6	Signals and sytems- A Nagoor Kani, Mc Graw Hill

Unit	**Topic**	**Chapters Nos** Book 1	Book 2	Book 3	Book 4	Book 5	Book 6	**No. of classes**
I	Introduction to signals and system				1		1	3
	Orthogonality concept in vectors and signals	3						4
	Various functions	10						3
II	Introduction to Fourier analysis	3			3		4	2
	Fourier series	3			3		4	4
	Fourier transform	4			5		4	5
	Hilbert Transform				7			2
III	LTI system	5			9		3	5
	Distortion less Transmission							3
	Concept of Convolution				10		7	4
IV	Laplace Transforms			6				5
	Z- Transforms			6				5
V	Sampling Theorem			7				4
	Correlation			7				4
	Spectral Density function			7				4

Contact classes for syllabus coverage	**57**
Lectures beyond syllabus	**02**
Tutorial classes	**09**
Classes for gaps& Add-on classes	**02**
Total number of classes	**70**

11. MID EXAM DESCRIPTIVE QUESTION PAPERS

K. G. Reddy College of Engineering &Technology

(Approved by AICTE, Affiliated to JNTUH)

Chilkur (Vil), Moinabad (Mdl), RR District

Name of the Exam: I Mid Examinations **September– 2018**

Year-Sem & Branch: II-I & ECE **Duration: 60 Min**

Subject:SSP **Date & Session:05-09-18(AN)**

Answer **ANY TWO** of the following Questions 2**X5=10**

Q.NO	QUESTION	Bloom's level	Course outcome
1	A rectangular function defined by $f(t)= 1\ ,\ 0<t<\pi$ $-1,\ \ \pi<t<2\pi$ Approximate above rectangular function by a single sinusoid sint, evaluate mean square error in this approximation.also show that what happens when more no. of sinusoidal are used for approximation.	Apply	CO2
2	a) Test the linearity,time invariant of the system governed by the equation i. $y(n)=ax(n)+b$ ii. $Y(t)=\log_{10} x(t)$ b) Obtain the conditions for the distortion less transmission through a system	Analyze	CO1
3	a) Expand the following function f(t) by trigonometric fourier series over the interval (0,1).In this interval f(t) is expressed as f(t)=At. b) State & prove any four properties of fourier transform	Apply & Remember	CO2
4	a) State and prove the sampling theorem b) Find the fourier transform of symmetrical gate pulse and sketch the spectrum.	Remember & Apply	CO2

K. G. Reddy College of Engineering &Technology

(Approved by AICTE, Affiliated to JNTUH)

Chilkur (Vil), Moinabad (Mdl), RR District

Name of the Exam: II Mid Examinations **November– 2018**

Year-Sem & Branch: II-I & ECE **Duration: 60 Min**

Subject:SSP **Date & Session:13/11/2018**

Answer **ANY TWO** of the following Questions 2**X5=10**

12. MID EXAM OBJECTIVE QUESTION PAPERS

Q.NO	QUESTION	Bloom's level	Course outcome
1	a) Find the z-transform and ROC of the signal $x(n)= -a^n u(-n-1)$ b) State and prove any four properties of z-transform	Apply & Remember	CO3
2	Show that the random process $X(t)= A \cos(w_0 t+\theta)$ is wide-sense stationary if it is assumed that A and w_0 are constants and θ is a uniformly density random variable over the interval $(0,2\Pi)$	Apply	CO4
3	Explain about auto correlation and cross correlation functions with their properties	Understand	CO4
4	Derive the relationship between power density spectrum and auto correlation function	Analyze	CO5

KG REDDY
College of Engineering
& Technology

SIGNALS & SYSTEMS (EC304PC)

B.TECH II Year I SEM I MID Term Examinations, SEP-2018

SIGNALS & STOCHASTIC PROCESSES

OBJECTIVE EXAM

Answer all the questions. All questions carry equal marks. Time: 20min. 10 marks.

I choose correct alternative:

1.	Two signals are orthogonal if the integral of their product is			[ ]
A.0	B.1	C.2	D.∞	
2.	The unit step signal can be obtained by ______ the ramp signal			[ ]
A. integrating	B.multiplying	C.differentiating	D.shifting	
3.	A exponential Fourier series has			[ ]
A. a two sided spectrum	B.a one sided spectrum	C.both a & b	D.None	
4.	The relation between unit step function and signum function is			[ ]
A. sgn(t)=2u(t)-1	B.sgn(t)=2u(t)+1	C.sgn(t)=2u(t)	D.None of these	
5.	The trigonometric Fourier series representation of a function with odd symmetry consists of			[ ]
A. sine terms only	B.odd harmonics	C.cosine terms only	D.even harmonics	

6.	For a distortion less transmission, the magnitude response |H(W)| is			[ ]
A. constant	B.zero	C.linear	D.infinite	
7.	The Fourier transform of unit impulse function is			[ ]
A.1	B.1/j	C.0	D.2Π ( )	
8.	The nyquist rate of a signal is			[ ]
A.$f_m/2$	B.f_m	C.$2f_m$	D.$f_m x f_m$	
9.	Convolution in time domain is equal to……………..in frequency domain			[ ]
A. differentiation	B.multiplication	C.addition	D.integration	
10.	If the sampling frequency of a signal is f_s=2000 samples per second ,the nyquist interval is			[ ]
A.5 ms	B.0.5 ms	C.0.5 seconds	D.5 seconds	

II Fill in the Blanks:

1.	If we approximate x(t) by a___________ number of orthogonal functions, the error become small
2.	The trigonometric Fourier series coefficient a_n in terms of exponential fourier series coefficient is ___________

3.	The magnitude line spectrum is always an ___________function of n
4.	The process of converting a continuous time signal into a discrete time signal is called___________
5.	The conditions to be satisfied for the Fourier series of a function to exist are called___________ conditions
6.	The Fourier transform of x(t) is given by___________
7.	The Fourier transform of $x(t-t_0)$ is___________
8.	The output of an LTI system is equal to impulse response when the input is___________
9.	The convolution of $x_1(t)$ & $x_2(t)$ can be expressed as ___________
10.	Aliasing occurs when___________

Key paper

I MULTIPLE CHOICE QUESTTIONS

1. A
2. C
3. A

4. A
5. A
6. A
7. A
8. C
9. B
10. B

II FILL IN THE BLANKS

1. Larger
2. $a_n = F_n + F_{-n}$
3. even
4. sampling
5. dirichlet's
6. $\int_{-\infty}^{\infty} x(t)e^{-jwt}\,dt$
7. $e^{-jwto}\,X(\omega)$
8. $unit\ impulse$
9. $\int_{-\infty}^{\infty} x_1(\tau)\,x_2(t-\tau)\,d\tau$
10. $f_s < 2f_m$

KG REDDY
College of Engineering
& Technology

SUBJECT CODE:EC304ES SET NO. 1

KG REDDY COLLEGE OF ENGINEERING & TECHNOLOGY

Chilkur (Vill) Moinabad (Mdl) R R Dist

B.TECH II Year I SEM II MID Term Examinations, NOV-2018

SIGNALS & STOCHASTIC PROCESS

OBJECTIVE EXAM

NAME____________________________HALL TICKET NO

		Q	M		A				

Answer all the questions. All questions carry equal marks. Time: 20min. 10 marks.

I choose correct alternative:

1.	If z-transform of x(n) is X(Z) , then the z-transform of x(-n) is		[ ]
A.X(-Z)	B.X(Z^{-1})	C.X(1-Z)	D.X(1+Z)
2.	Which of the following method is used to obtain inverse Z-transform		[ ]
A. power series method	B. partial fraction method	C. residue method	D. all
3.	If the future value of the sample function can be predicted based on its past values, the process is referred to as		[ ]
A .deterministic process	B. non-deterministic process	C. independent process	D. statistical process
4.	A process is said to be continuous random sequence		[ ]
A.If the random variable X is continuous and time t is continuous	B. If the random variable X is discrete and time t is discrete	C. If the random variable X is continuous and time t is discrete	D. If the random variable X is discrete and time t is continuous
5.	$R_{XY}(\tau)=R_{YX}(-\tau)$ is a		[ ]
A. symmetry property	B.periodicity property	C. asymmetry property	D. ergodic property
6.	A process is said to be an ergodic process if		[ ]
A. time averages are zero	B. ensemble averages are zero	C. time averages are infinity	D. time averages are equal to ensemble averages
7.	The auto correlation function of X(t), $R_{XX}(\tau)$ is		[ ]
A. E[X^2(t)]	B.$\int_{-\infty}^{\infty} X(t)dt$	C. $\int_{\infty}^{\infty} X(2t)dt$	D. E[X(t) X(t+ τ)]
8.	The time average of auto correlation function and the power spectral density		[ ]

	form a pair of			
A. z-transform	B. fourier transform	C. laplace transform	D. convolution	
9.	For a given random process X(t), the mean value is 6 and auto correlation function is $R_{XX}(\tau)= 36+25\, e^{-\lvert\tau\rvert}$, the average power is			[ ]
A.16	B.6	C.54	D. 61	
10.	The cross correlation between X(t) and Y(t) is $R_{XY}(\tau)=$			[ ]
A. $h(\tau)*R_{XX}(\tau)$	B. $h(-\tau)*R_{XX}(\tau)$	C. $h(-\tau)*R_{XY}(\tau)$	D. $h(\tau)*R_{YX}(\tau)$	

Count…..2

SUBJECT CODE:EC304ES SET NO. 1

II Fill in the Blanks:

1.	ROC does not contain any_______________
2.	Z-transform of a^n x(n) is___________
3.	A system is stable if ROC ____
4.	The _______________process is used to describe the number of times the event occurred as a function of time
5.	Power spectral density is always__________ and ____________
6.	If a random process is stationary to all orders n=1,2,3------N, it is called__________stationarity
7.	At τ =0, the auto covariane function becomes the _______________of the random process
8.	If the processes X(t) and Y(t) are orthogonal, then δ_{XY} (w) is _______________
9.	For a WSS process, psd at zero frequency gives____________
10.	$R_{XX}(\tau)$ is an _________ function of τ

Key paper

I MULTIPLE CHOICE QUESTTIONS

1. A
2. D
3. A
4. C
5. A
6. D
7. D
8. B
9. D
10. A

II FILL IN THE BLANKS

1. Poles
2. X(z/a)
3. Within the unit circle
4. Poisson
5. Real and negative
6. SSS
7. Variance
8. Zero
9. Area under the graph of autocorrelation process
10. even

13. ASSIGNMENT TOPICS WITH MATERIALS

UNIT-I

1. Signal approximation using Orthogonal functions

Let us consider a set of n mu(t)ually orthogonal functions $x_1 t$, $x_2 t$... $x_n t$ over the interval t_1 to t_2. As these functions are orthogonal to each other, any two signals $x_{j(t)}$, $x_{k(t)}$ have to satisfy the orthogonality condition. i.e.

$$\int_{t_1}^{t_2} x_j(t)x_k(t)dt = 0 \text{ where } j \neq k$$

$$\text{Let} \int_{t_1}^{t_2} x_k^2(t)dt = k_k$$

Let a function ft, it can be approximated with this orthogonal signal space by adding the components along mu(t)ually orthogonal signals i.e.

$$f(t) = C_1x_1(t) + C_2x_2(t) + \ldots + C_nx_n(t) + f_e(t)$$

$$= \Sigma_{r=1}^{n} C_r x_r(t)$$

$$f(t) = f(t) - \Sigma_{r=1}^{n} C_r x_r(t)$$

Mean sqaure error

$$\varepsilon = \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f_e(t)]^2 dt$$

$$= \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f[t] - \sum_{r=1}^{n} C_r x_r(t)]^2 dt$$

The component which minimizes the mean square error can be found by

$$\frac{d\varepsilon}{dC_1} = \frac{d\varepsilon}{dC_2} = \ldots = \frac{d\varepsilon}{dC_k} = 0$$

Let us consider $\frac{d\varepsilon}{dC_k} = 0$

$$\frac{d}{dC_k}[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f(t) - \Sigma_{r=1}^{n} C_r x_r(t)]^2 dt] = 0$$

All terms that do not contain C_k is zero. i.e. in summation, r=k term remains and all other terms are zero.

$$\int_{t_1}^{t_2} -2f(t)x_k(t)dt + 2C_k \int_{t_1}^{t_2} [x_k^2(t)]dt = 0$$

$$\Rightarrow C_k = \frac{\int_{t_1}^{t_2} f(t)x_k(t)dt}{int_{t_1}^{t_2} x_k^2(t)dt}$$

$$\Rightarrow \int_{t_1}^{t_2} f(t)x_k(t)dt = C_k K_k$$

2. Mean square error

The average of square of error function f t is called as mean square error. It is denoted by ε *epsilon* .

$$\varepsilon = \frac{1}{t_2-t_1} \int_{t_1}^{t_2} [f_e(t)]^2 dt$$

$$= \frac{1}{t_2-t_1} \int_{t_1}^{t_2} [f_e(t) - \Sigma_{r=1}^{n} C_r x_r(t)]^2 dt$$

$$= \frac{1}{t_2-t_1} [\int_{t_1}^{t_2} [f_e^2(t)]dt + \Sigma_{r=1}^{n} C_r^2 \int_{t_1}^{t_2} x_r^2(t)dt - 2\Sigma_{r=1}^{n} C_r \int_{t_1}^{t_2} x_r(t)f(t)dt$$

You know that $C_r^2 \int_{t_1}^{t_2} x_r^2(t)dt = C_r \int_{t_1}^{t_2} x_r(t)f(d)dt = C_r^2 K_r$

$$\varepsilon = \frac{1}{t_2-t_1} [\int_{t_1}^{t_2} [f^2(t)]dt + \Sigma_{r=1}^{n} C_r^2 K_r - 2\Sigma_{r=1}^{n} C_r^2 K_r]$$

$$= \frac{1}{t_2-t_1} [\int_{t_1}^{t_2} [f^2(t)]dt - \Sigma_{r=1}^{n} C_r^2 K_r]$$

$$\therefore \varepsilon = \frac{1}{t_2-t_1} [\int_{t_1}^{t_2} [f^2(t)]dt + (C_1^2 K_1 + C_2^2 K_2 + \ldots + C_n^2 K_n)]$$

3. Linear Time Invariant (LTI) System

A system is said to be linear when it satisfies superposition and homogenate principles. Consider two systems with inputs as x1(t) , x2(t) , and outputs as y1(t) , y2(t) respectively. Then, according to the superposition and homogenate principles,

T [a1 x1(t) + a2 x2(t)] = a1 T[x1(t)] + a2 T[x2(t)] ∴, T [a1 x1(t) + a2 x2(t)] = a1 y1(t) + a2 y2(t)

From the above expression, is clear that response of overall system is equal to response of individual system.

Example:

t = x2(t)

Solution:

$y1\ (t) = T[x1(t)] = x12(t)$

$y2\ (t) = T[x2(t)] = x22(t)$

$T\ [a1\ x1(t) + a2\ x2(t)] = [\ a1\ x1(t) + a2\ x2(t)]2$

Which is not equal to a1 y1(t) + a2 y2(t). Hence the system is said to be non linear.

A system is said to be time variant if its input and output characteristics vary with time. Otherwise, the system is considered as time invariant.

The condition for time invariant system is:
$x(t-t_0) = y(t-t_0)$

The condition for time variant system is:

$x(t-t_0) \neq y(t-t_0)$

Example:
$y(t)=2x(t)$

If a system is both liner and time variant, then it is called liner time variant *LTV* system.

If a system is both liner and time Invariant then that system is called liner time invariant *LTI* system.

4. Ideal LPF, HPF and BPF characteristics

An ideal frequency reflective filter passes complex exponential signal. for a given set of frequencies and completely rejects the others. Figure (9.1) shows frequency response for ideal low pass filter (LPF), ideal high pass filter (HPF), ideal bandpass filter (BPF) and ideal backstop filter (BSF).

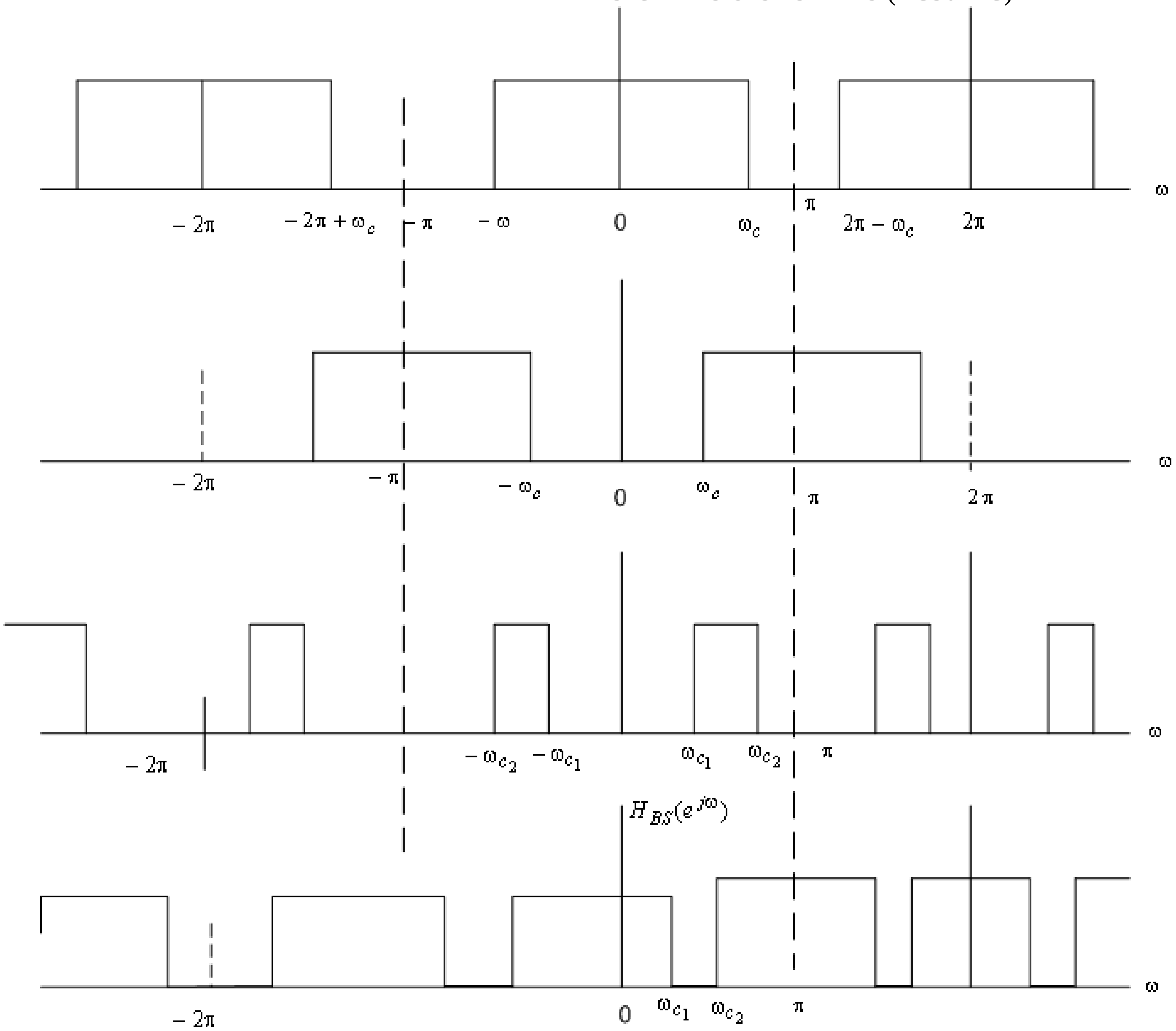


Fig 9.1

The ideal filters have a frequency response that is real and non-negative, in other words, has a zero phase characteristics. A linear phase characteristics introduces a time shift and this causes no distortion in the shape of the signal in the passband.

Since the Fourier transfer of a stable impulse response is continuous function of ω, can not get a stable ideal filter.

5. Graphical representation of Convolution

Steps for Graphical Convolution x (t)* h (t)

1. Re-Write the signals as functions of τ: x (τ) and h (τ)

2. Flip just one of the signals around t = 0 to get either x(- τ) or h(- τ) a. It is usually best to flip the signal with shorter duration b. For notational purposes here: we'll flip h (τ) to get h(- τ)

3. Find Edges of the flipped signal a. Find the left-hand-edge τ-value of h(- τ): call it τ L,0 b. Find the right-hand-edge τ-value of h(- τ): call it τ R,0

4. Shift h(- τ) by an arbitrary value of t to get h (t - τ) and get its edges a. Find the left-hand-edge τ-value of h (t - τ) as a function of t: call it τL,t • Important: It will always be… τL,t = t + τ L,0 b. Find the right-hand-edge τ-value of h (t - τ) as a function of t: call it τR,t • Important: It will always be… τR,t = t + τ R,0

Note: I use τ for what the book uses λ... It is not a big deal as they are just dummy variables!!! =

5.Find Regions of τ-Overlap a. What you are trying to do here is find intervals of t over which the product x(τ) h(t - τ) has a single mathematical form in terms of τ b. In each region find: Interval of t that makes the identified overlap happen

6.For Each Region: Form the Product x(τ) h(t - τ) and Integrate a. Form product x(τ) h(t - τ) b. Find the Limits of Integration by finding the interval of τ over which the product is nonzero i. Found by seeing where the edges of x(τ) and h(t - τ) lie ii. Recall that the edges of h(t - τ) are τL,t and τR,t , which often depend on the value of t • So… the limits of integration may depend on t c. Integrate the product x(τ) h(t - τ) over the limits found in 6b i. The result is generally a function of t, but is only valid for the interval of t found for the current region ii. Think of the result as a "time-section" of the output y(t)

UNIT-II

1. Representation of Fourier series

A signal is said to be periodic if it satisfies the condition x t = x $t + T$ or x n = x $n + N$.

Where T = fundamental time period,

ω_0= fundamental frequency = $2\pi/T$

There are two basic periodic signals:

$x(t) = \cos \omega_0 t$ *sinusoidal* &

$x(t) = e^{j\omega_0 t}$ *complex exponential*

These two signals are periodic with period $T = 2\pi/\omega_0$.

A set of harmonically related complex exponentials can be represented as $\{\phi_k(t)\}$

$$\phi_k(t) = \{e^{jk\omega_0 t}\} = \{e^{jk(\frac{2\pi}{T})t}\} \text{where } k = 0 \pm 1, \pm 2..n \ \ ..... (1)$$

All these signals are periodic with period T

According to orthogonal signal space approximation of a function x t with n, mutually orthogonal functions is given by

$x(t) = \sum a_k e^{jk\omega_0 t} ..... (2)$

$$\sum_{k=-\infty}^{\infty} a_k k e^{jk\omega_0 t}$$

Where a_k= Fourier coefficient = coefficient of approximation. This signal xt is also periodic with period T.

Equation 2 represents Fourier series representation of periodic signal xt.

The term k = 0 is constant.

The term $k = \pm 1$ having fundamental frequency ω_0, is called as 1st harmonics.

The term $k = \pm 2$ having fundamental frequency $2\omega_0$, is called as 2nd harmonics, and so on...

The term $k = \pm n$ having fundamental frequency $n\omega_0$, is called as nth harmonics.

2. **Trigonometric Fourier Series**

$\sin n\omega_0 t$ and $\sin m\omega_0 t$ are orthogonal over the interval $(t_0, t_0 + \frac{2\pi}{\omega_0})$. So $\sin \omega_0 t$, $\sin 2\omega_0 t$ forms an orthogonal set. This set is not complete without $\{\cos n\omega_0 t\}$ because this cosine set is also orthogonal to sine set. So to complete this set we must include both cosine and sine terms. Now the complete orthogonal set contains all cosine and sine terms i.e. {sin $n\omega 0$ t, cos $n\omega 0$ t } where n=0, 1, 2...

$\therefore$ Any function xt in the interval $(t_0, t_0 + \frac{2\pi}{\omega_0})$ can be represented as

$$x(t) = a_0 \cos 0\omega_0 t + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \ldots + a_n \cos n\omega_0 t + \ldots$$
$$+ b_0 \sin 0\omega_0 t + b_1 \sin 1\omega_0 t + \ldots + b_n \sin n\omega_0 t + \ldots$$
$$= a_0 + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \ldots + a_n \cos n\omega_0 t + \ldots$$
$$+ b_1 \sin 1\omega_0 t + \ldots + b_n \sin n\omega_0 t + \ldots$$

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad (t_0 < t < t_0 + T)$$

The above equation represents trigonometric Fourier series representation of xt.

$$\text{Where } a_0 = \frac{\int_{t_0}^{t_0+T} x(t) \cdot 1 dt}{\int_{t_0}^{t_0+T} 1^2 dt} = \frac{1}{T} \cdot \int_{t_0}^{t_0+T} x(t) dt$$

$$a_n = \frac{\int_{t_0}^{t_0+T} x(t) \cdot \cos n\omega_0 t \, dt}{\int_{t_0}^{t_0+T} \cos^2 n\omega_0 t \, dt}$$

$$b_n = \frac{\int_{t_0}^{t_0+T} x(t) \cdot \sin n\omega_0 t \, dt}{\int_{t_0}^{t_0+T} \sin^2 n\omega_0 t \, dt}$$

$$\text{Here } \int_{t_0}^{t_0+T} \cos^2 n\omega_0 t \, dt = \int_{t_0}^{t_0+T} \sin^2 n\omega_0 t \, dt = \frac{T}{2}$$

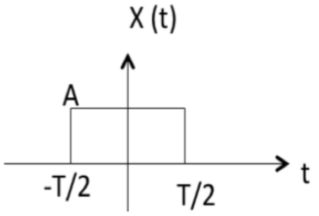
$$\therefore a_n = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \cos n\omega_0 t \, dt$$

$$b_n = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \sin n\omega_0 t \, dt$$

3. Fourier Transform of standard signals

FT of GATE Function

Fourier Transform of Basic Functions



$$F[\omega] = ATSa(\frac{\omega T}{2})$$

FT of Impulse Function

$FT[\omega(t)] = [\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt] = e^{-j\omega t} |_{t = 0}$

$= e^0 = 1$

$\therefore \delta(\omega) = 1$

FT of Unit Step Function:

$U(\omega) = \pi\delta(\omega) + 1/j\omega$

FT of Exponentials

$e^{-at}u(t) \overset{F.T}{\longleftrightarrow} 1/(a + j\omega)$

$e^{-at}u(t) \overset{F.T}{\longleftrightarrow} 1/(a + j\omega)$

$e^{-a|t|} \overset{F.T}{\longleftrightarrow} \frac{2a}{a^2+\omega^2}$

$e^{j\omega_0 t} \overset{F.T}{\longleftrightarrow} \delta(\omega - \omega_0)$

FT of Signum Function

$sgn(t) \overset{F.T}{\longleftrightarrow} 2/ jw$

4. Properties of Fourier Transform

Here are the properties of Fourier Transform:

a. Linearity Property

If $F.T[x(t)] \longleftrightarrow X(\omega)$

& $F.T[y(t)] \longleftrightarrow Y(\omega)$

Then linearity property states that

$ax(t) + by(t) \overset{F.T}{\longleftrightarrow} aX(\omega) + bY(\omega)$

b. Time Shifting Property

If F.T[$x(t)$] $\leftrightarrow X(\omega)$

Then Time shifting property states that

$F.T[x(t-t_0)] \leftrightarrow e^{-j\omega t_0} X(\omega)$

c. Frequency Shifting Property

If F.T[$x(t)$] $\leftrightarrow X(\omega)$

Then frequency shifting property states that

$F.T[e^{j\omega_0 t}.x(t)] \leftrightarrow X(\omega-\omega_0)$

d. Time Reversal Property

If F.T[x (t)] $\leftrightarrow X(\omega)$

Then Time reversal property states that

$F.T[x(-t)] \leftrightarrow X(-\omega)$

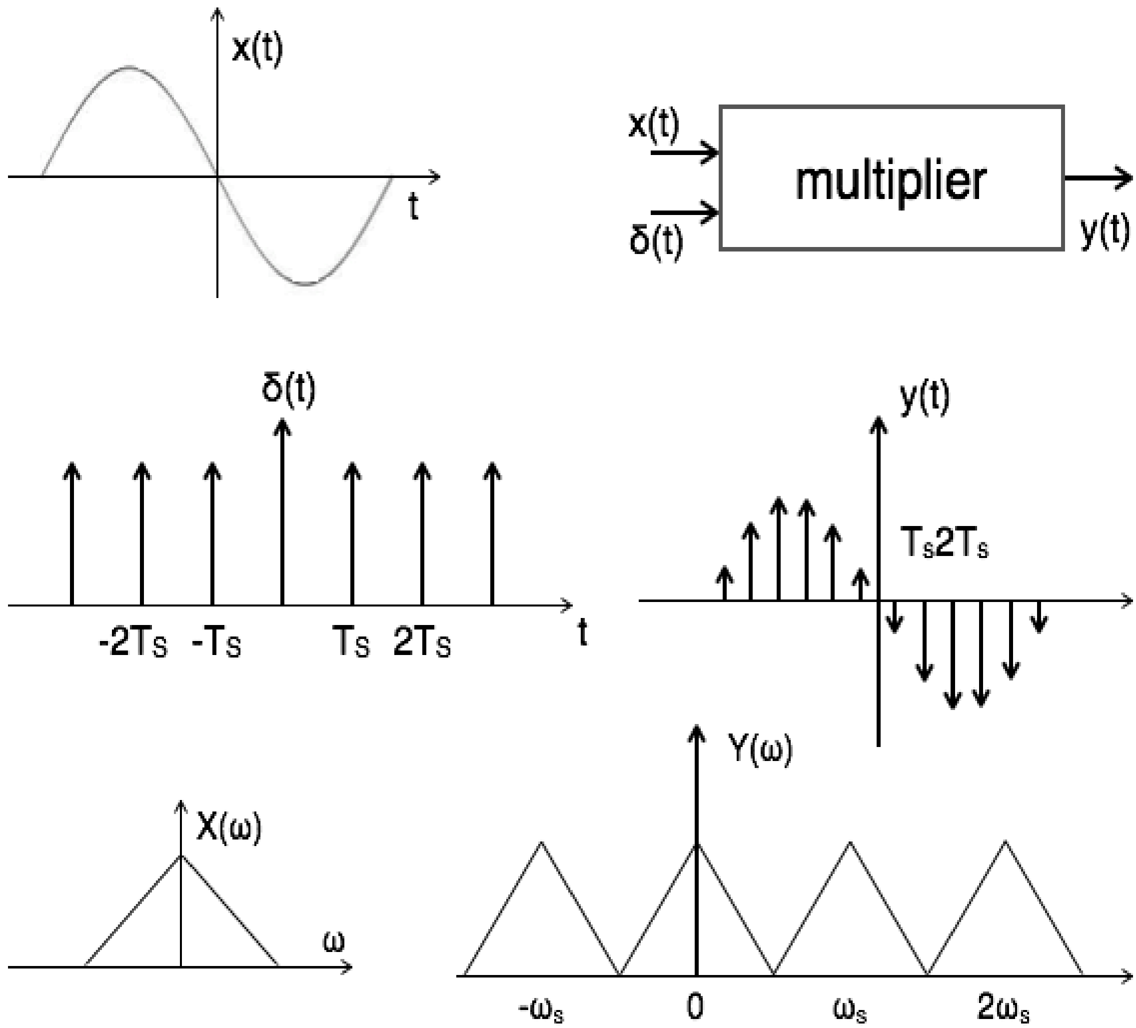
5. Sampling theorem

Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e.

$$F_s \geq 2f_m$$

Proof: Consider a continuous time signal x(t). The spectrum of x(t) is a band limited to f_m Hz i.e. the spectrum of x(t) is zero for $|\omega| > \omega_m$.

Sampling of input signal x(t) can be obtained by multiplying x(t) with an impulse train δ(t) of period T_s. The output of multiplier is a discrete signal called sampled signal which is represented with y(t) in the following diagrams:



Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

Take Fourier transform on both sides.

$$\text{Sampled signal } y(t) = x(t).\,\delta(t) \;\ldots\ldots(1)$$

The trigonometric Fourier series representation of δt is given by

$$\delta(t) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos n\omega_s t + b_n \sin n\omega_s t) \;\ldots\ldots(2)$$

Where $a_0 = \frac{1}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)dt = \frac{1}{T_s}\delta(0) = \frac{1}{T_s}$

$$a_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\cos n\omega_s\, dt = \frac{2}{T_2}\delta(0)\cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\sin n\omega_s t\, dt = \frac{2}{T_s}\delta(0)\sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore\; \delta(t) = \frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t + 0)$$

Substitute δt in equation 1.

$$\rightarrow y(t) = x(t).\,\delta(t)$$

$$= x(t)[\frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t)]$$

$$= \frac{1}{T_s}[x(t) + 2\Sigma_{n=1}^{\infty}(\cos n\omega_s t)x(t)]$$

$$y(t) = \frac{1}{T_s}[x(t) + 2\cos\omega_s t.\,x(t) + 2\cos 2\omega_s t.\,x(t) + 2\cos 3\omega_s t.\,x(t)\ldots\ldots]$$

Take Fourier transform on both sides.

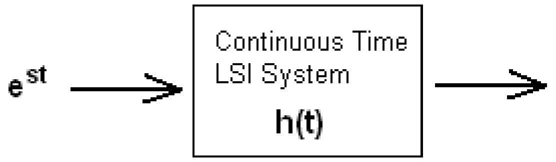
$$Y(\omega) = \frac{1}{T_s}[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s)+\ldots]$$

$$\therefore\; Y(\omega) = \frac{1}{T_s}\Sigma_{n=-\infty}^{\infty}X(\omega - n\omega_s) \qquad where\ n = 0, \pm1, \pm2, \ldots$$

UNIT-III

1. Review of Laplace Transforms

The response of a Linear Time Invariant system with impulse response **h(t)** to a complex exponential input of the form $\mathbf{e^{st}}$ can be represented in the following way :



$$\int_{-\infty}^{+\infty} h(\lambda)e^{s(t-\lambda)}d\lambda$$

$$= e^{st}\int_{-\infty}^{+\infty} h(\lambda)e^{-s\lambda}d\lambda$$

Let $$H(s)=\int_{-\infty}^{+\infty} h(\lambda)e^{-s\lambda}d\lambda$$

Where **H(s)** is known as the Laplace Transform of **h(t)**. We notice that the limits are from [-∞ to +∞] and hence this transform is also referred to as **Bilateral or Double sided** Laplace Transform. There exists a one-to-one correspondence between the **h(t)** and **H(s)** i.e. the original domain and the transformed domain. Therefore L.T. is a unique transformation and the 'Inverse Laplace Transform' also exists.

Note that $\mathbf{e^{st}}$ is also an **eigen function** of the LSI system only if **H(s) converges**. The range of values for which the expression described above is finite is called as the Region of Convergence (**ROC**). In this case, the region of convergence is **Re(s)** > **0**.

Thus, the Laplace transform has two parts which are , the expression and region of convergence respectively. The region of convergence of the Laplace transform is essentially determined by **Re(s)**

2. Properties of Laplace Transforms

The properties of Laplace transform are:

1) Linearity Property

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

& $y(t) \overset{L.T}{\longleftrightarrow} Y(s)$

Then linearity property states that

$$ax(t) + by(t) \overset{L.T}{\longleftrightarrow} aX(s) + bY(s)$$

2) Time Shifting Property

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then time shifting property states that

$$x(t - t_0) \overset{L.T}{\longleftrightarrow} e^{-st_0} X(s)$$

3) Frequency Shifting Property

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then frequency shifting property states that

$$e^{s_0 t}.x(t) \overset{L.T}{\longleftrightarrow} X(s - s_0)$$

4) Time Reversal Property

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then time reversal property states that

$$x(-t) \overset{L.T}{\longleftrightarrow} X(-s)$$

5) Time Scaling Property

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then time scaling property states that

$$x(at) \overset{L.T}{\longleftrightarrow} \frac{1}{|a} X(\frac{s}{a})$$

3. Relation between L.T and F.T of a signal

The Fourier Transform for Continuous Time signals is infact a special case of Laplace Transform. This fact and subsequent relation between LT and FT are explained below.

Now we know that Laplace Transform of a signal 'x'(t)' is given by:

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

The s-complex variable is given by $s = \sigma + j\Omega$

But we consider $\sigma = 0$ and therefore 's' becomes completely imaginary. Thus we have $s = j\Omega$. This means that we are only considering the vertical strip at $\sigma = 0$.

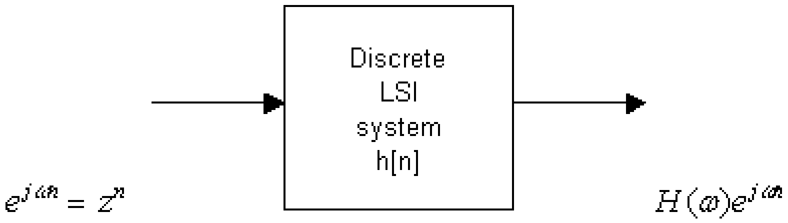
$$X(j\Omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\Omega t}dt$$

From the above discussion it is clear that the LT reduces to FT when the complex variable only consists of the imaginary part . Thus LT reduces to FT along the $j\Omega$ -axis (Imaginary axis).

$$\text{Fourier Transform of } x(t) = \text{Laplace Transform of } x(t)\,|_{s=j\Omega}$$

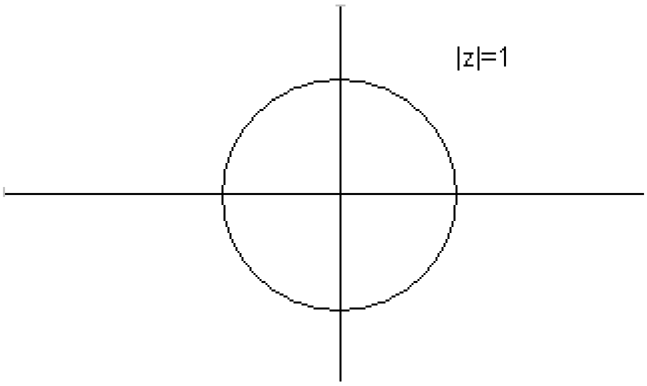
4. Concept of Z-Transform of a Discrete Sequence

The response of a linear time-invariant system with impulse response h[n] to a complex exponential input of the form z^n can be represented in the following way :



where $H(\omega) = \sum h[n]e^{-j\omega n}$

In the complex z-plane , we take a circle with unit radius centered at the origin.



$\Rightarrow z = e^{j\omega}$
is a unit circle
in z-plane

H(w) is periodic with period 2Π with respect to ' w ' .

When we replace **z** by $e^{j\omega}$,we get periodicity of 2Π in the form of a circle.

5. Inverse Z-transform

We know that there is a one to one correspondence between a sequence **x[n]** and its ZT which is **X[z]**.
Obtaining the sequence 'x[n]' when 'X[z]' is known is called Inverse Z - Transform.
For a ready reference , the ZT and IZT pair is given below.

X[z] = Z { x[n] } Forward Z - Transform
x[n] = Z^{-1} { X[z] } Inverse Z - Transform

For a discrete variable signal x[n], if its z - Transform is X(z), then the inverse z - Transform of X(z) is given by

$$\boxed{x[n] = \frac{1}{2\pi j}\oint_C X(z) z^{(n-1)} dz}$$

where ' **C** ' is any closed contour which encircles the origin and lies ENTIRELY in the Region of Convergence.

UNIT-IV

1. Random Process Concept

A Random Variable ‘X’ is defined as a function of the possible outcomes ‘**s ’** of an experiment or whose value is unknown and possibly depends on a set of random events. It is denoted by X(s).

The Concept of Random Process is based on enlarging the random variable concept to include time ‘t’ and is denoted by X(t,s) i.e., we assign a time function to every outcome according to some rule. In short, it is represented as X(t). A random process clearly represents a family or ensemble of time functions when t and s are variables. Each member time function is called a sample function or ensemble member.

Depending on time‘t’ and outcome’ s’ fixed or variable, A random process represents a single time function when t is a variable and s is fixed at a specific value. A random process represents a random variable when t is fixed and s is a variable a random process represents a number when t and s are both fixed

2. Distribution and Density Functions

Distribution Function:

Probability distribution function (PDF) which is also be called as Cumulative Distribution Function (CDF) of a real valued random variable ‘X ‘ is the probability that X will take value less than or equal to X.

It is given by

$$F_X\ (x) = \ P\{X\ \leq x\}$$

In case of random process X(t), for a particular time t, the distribution function associated with the random variable X is denoted as

$$F_X\ (x{:}\, t) = \ P\{X(t)\ \leq x\}$$

In case of two random variables, X1 = X(t1) and X2 = X (t2), the second order joint distribution function is two dimensional and given by

$$F_X\ \left(x_{1,}, x_2 : t_{1,}, t_2\right) = \ P\{X(t_1)\ \leq x_{1,}, X(t_2)\ \leq x_2\ \}$$

and can be similarly extended to N random variables, called as Nth order joint distribution function

Density Function:

The probability density function(pdf) in case of random variable is defined as the derivative of the distribution function and is given by

$$f_X\ (x) = \frac{dF_x\ (x)}{dx}$$

In case of random process, density function is given by

$$f_X\ (x\!:t) = \frac{dF_x\ (x\!:t)}{dx}$$

In case of two random functions, two dimensional density function is given by

$$f_X\ \left(x_{1,}, x_2\ :t_{1,}, t_2\right) = \frac{\partial^2 F_X\ \left(x_{1,}, x_2\ :t_{1,}, t_2\right)}{\partial x_1 \partial x_2}$$

3. Time Averages and Ergodicity

Time Average Function: Consider a random process X(t). Let x(t) be a sample function which exists for all time at a fixed value in the given sample space S. The average value of x(t) taken over all times is called the time average of x(t). It is also called mean value of x(t).

It can be expressed as. $\bar{x} = \mathrm{A}[\mathrm{x(t)}] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)dt.$

Time autocorrelation function: Consider a random process X(t). The time average of the product X(t) and X(t+ τ) is called time average autocorrelation function of x(t) and is denoted

as $R_{xx}(\tau) = A[X(t)\ X(t+\tau)]$ or $R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)dt.$

Time mean square function: If τ = 0, the time average of $x^2(t)$ is called time mean square value of x(t) defined as = $A[X^2(t)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x^2(t)dt.$

Time cross correlation function: Let X(t) and Y(t) be two random processes with sample functions x(t) and y(t) respectively. The time average of the product of x(t) y(t+ τ) is called time cross correlation function of x(t) and y(t). Denoted as

$$\mathcal{R}_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)y(t+\tau)dt.$$

Ergodic Theorem and Ergodic Process: The Ergodic theorem states that for any random process X(t), all time averages of sample functions of x(t) are equal to the corresponding statistical or ensemble averages of X(t). i.e. $\bar{x} = \bar{X}$ or $\mathcal{R}_{xx}(\tau) = R_{XX}(\tau)$. Random processes that satisfy the Ergodic theorem are called Ergodic processes.

Joint Ergodic Process: Let X(t) and Y(t) be two random processes with sample functions x(t) and y(t) respectively. The two random processes are said to be jointly Ergodic if they are individually Ergodic and their time cross correlation functions are equal to their respective statistical cross correlation functions. i.e. 1. $\bar{x} = \bar{X}$, $\bar{y} = \bar{Y}$ 2. $\mathcal{R}_{xx}(\tau) = R_{XX}(\tau)$, $\mathcal{R}_{xy}(\tau) = R_{XY}(\tau)$ and $\mathcal{R}_{yy}(\tau) = R_{YY}(\tau)$.

Mean Ergodic Random Process: A random process X(t) is said to be mean Ergodic if time average of any sample function x(t) is equal to its statistical average, $\bar{X}$ which is constant and the probability of all other sample functions is equal to one. i.e. $E[X(t)] = \bar{X} = A[x(t)] = \bar{x}$ with probability one for all x(t).

Autocorrelation Ergodic Process: A stationary random process X(t) is said to be Autocorrelation Ergodic if and only if the time autocorrelation function of any sample function x(t) is equal to the statistical autocorrelation function of X(t). i.e. A[x(t) x(t+τ)] = E[X(t) X(t+τ)] or $\mathcal{R}_{xx}(\tau) = R_{XX}(\tau)$.

Cross Correlation Ergodic Process: Two stationary random processes X(t) and Y(t) are said to be cross correlation Ergodic if and only if its time cross correlation function of sample functions x(t) and y(t) is equal to the statistical cross correlation function of X(t) and Y(t). i.e. A[x(t) y(t+τ)] = E[X(t) Y(t+τ)] or $\mathcal{R}_{xy}(\tau) = R_{XY}(\tau)$.

c) Autocorrelation Function and Its Properties

Properties of Autocorrelation function: Consider that a random process X(t) is at least WSS and is a function of time difference $\tau = t_2 - t_1$. Then the following are the properties of the autocorrelation function of X(t).

1. Mean square value of X(t) is $E[X^2(t)] = R_{XX}(0)$. It is equal to the power (average) of the process, X(t).
   Proof: We know that for X(t), $R_{XX}(\tau) = E[X(t)\,X(t+\tau)]$. If $\tau = 0$, then $R_{XX}(0) = E[X(t)\,X(t)] = E[X^2(t)]$ hence proved.
2. Autocorrelation function is maximum at the origin i.e. $|R_{XX}(\tau)| \le R_{XX}(0)$.
   Proof: Consider two random variables $X(t_1)$ and $X(t_2)$ of X(t) defined at time intervals t_1 and t_2 respectively. Consider a positive quantity $[X(t_1) \pm X(t_2)]^2 \ge 0$
   Taking Expectation on both sides, we get $E[X(t_1) \pm X(t_2)]^2 \ge 0$
   $E[X^2(t_1) + X^2(t_2) \pm 2X(t_1)\,X(t_2)] \ge 0$
   $E[X^2(t_1)] + E[X^2(t_2) \pm 2E[X(t_1)\,X(t_2)] \ge 0$
   $R_{XX}(0) + R_{XX}(0) \pm 2\,R_{XX}(t_1,t_2) \ge 0$ [Since $E[X^2(t)] = R_{XX}(0)$]
   Given X(t) is WSS and $\tau = t_2 - t_1$.
   Therefore $2\,R_{XX}(0 \pm 2\,R_{XX}(\tau) \ge 0$
   $R_{XX}(0 \pm R_{XX}(\tau) \ge 0$ or
   $|R_{XX}(\tau)| \le R_{XX}(0)$ hence proved.
3. $R_{XX}(\tau)$ is an even function of τ i.e. $R_{XX}(-\tau) = R_{XX}(\tau)$.
   Proof: We know that $R_{XX}(\tau) = E[X(t)\,X(t+\tau)]$
   Let $\tau = -\tau$ then
   $R_{XX}(-\tau) = E[X(t)\,X(t-\tau)]$
   Let $u = t - \tau$ or $t = u + \tau$
   Therefore $R_{XX}(-\tau) = E[X(u+\tau)\,X(u)] = E[X(u)\,X(u+\tau)]$

d) Gaussian Random Processes

Gaussian Random Process: Consider a continuous random process X(t). Let N random variables $X_1=X(t_1), X_2=X(t_2), \ldots, X_N=X(t_N)$ be defined at time intervals $t_1, t_2, \ldots t_N$ respectively. If random variables are jointly Gaussian for any N=1,2,…. And at any time

instants $t_1, t_2, \ldots t_N$. Then the random process X(t) is called Gaussian random process. The Gaussian density function is given as

$$f_X(x_1, x_2 \ldots\ldots x_N ; t_1, t_2 \ldots\ldots t_N) = \frac{1}{(2\pi)^{N/2}|[C_{XX}]|^{1/2}}\, exp(-[X-\overline{X}]^T[C_{XX}]^{-1}[X-\overline{X}])/2$$

where C_{XX} is a covariance matrix.

UNIT-V

1. Power Spectrum: Properties

Power Density Spectrum: The power spectrum of a WSS random process X (t) is defined as the Fourier transform of the autocorrelation function $R_{XX}(\tau)$ of X (t). It can be expressed as

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$$

We can obtain the autocorrelation function from the power spectral density by taking the inverse Fourier transform i.e

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega\tau}\, d\omega$$

Therefore, the power density spectrum $S_{XX}(\omega)$ and the autocorrelation function $R_{XX}(\tau)$ are Fourier transform pairs.

The power spectral density can also be defined as

$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$$

Where $X_T(\omega)$ is a Fourier transform of X(t) in interval [-T,T]

Properties of power density spectrum: The properties of the power density spectrum $S_{XX}(\omega)$ for a WSS random process X(t) are given as

(1) $S_{XX}(\omega) \geq 0$

Proof: From the definition, the expected value of a non negative function $E[\left||X_{T(\omega)}|^2\right|]$ is always non-negative.

Therefore $S_{XX}(\omega) \geq 0$ hence proved.

(2) The power spectral density at zero frequency is equal to the area under the curve of the autocorrelation $R_{XX}(\tau)$ i.e. $S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, d\tau$

Proof: From the definition we know that $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$ at $\omega=0$,

$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, d\tau$ hence proved

(3) The power density spectrum of a real process X(t) is an even function i.e.

$S_{XX}(-\omega) = S_{XX}(\omega)$

Proof: Consider a WSS real process X(t). then

$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$ also $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$

Substitute $\tau = -\tau$ then

$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau)\, e^{-j\omega\tau}\, d\tau$

Since X(t) is real, from the properties of autocorrelation we know that, $R_{XX}(-\tau) = R_{XX}(\tau)$

Therefore $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$

$S_{XX}(-\omega) = S_{XX}(\omega)$ hence proved.

(4) $S_{XX}(\omega)$ is always a real function

Proof: We know that $S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$

Since the function $\left||X_{T(\omega)}|^2\right|$ is a real function, $S_{XX}(\omega)$ is always a real function hence proved.

(5) If $S_{XX}(\omega)$ is a psd of the WSS random process X(t), then

$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = A\,\{E[X^2(t)]\} = R_{XX}(0)$ or The time average of the mean square value of a WSS random process equals the area under the curve of the power spectral density.

Proof: We know that $R_{XX}(\tau) = A\,\{E[X(t+\tau)\,X(t)]\}$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega$ at $\tau = 0$,

$R_{XX}(0) = A\{E[X^2(t)]\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega$ = Area under the curve of the power spectral density. Hence proved.

2. The Cross-Power Density Spectrum

Cross power density spectrum: Consider two real random processes X(t) and Y(t). which are jointly WSS random processes, then the cross power density spectrum is defined as the Fourier transform of the cross correlation function of X(t) and Y(t).and is expressed as

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$ and $S_{YX}(\omega) = \int_{-\infty}^{\infty} R_{YX}(\tau)\, e^{-j\omega\tau}\, d\tau$ by inverse Fourier transformation, we can obtain the cross correlation functions as

$R_{XY}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega)\, e^{j\omega\tau}\, d\omega$ and $R_{YX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{YX}(\omega)\, e^{j\omega\tau}\, d\omega$

Therefore the cross psd and cross correlation functions are forms a Fourier transform pair.

If $X_T(\omega)$ and $Y_T(\omega)$ are Fourier transforms of X(t) and Y(t) respectively in interval [-T,T], Then the cross power density spectrum is defined as

$S_{XY}(\omega) = \lim_{T\to\infty} \frac{E[\left|X_T(\omega) Y_T(\omega)\right|^1]}{2T}$ and $S_{YX}(\omega) = \lim_{T\to\infty} \frac{E[\left|Y_T(\omega) X_T(\omega)\right|^1]}{2T}$

3. Relationship between Power density Spectrum and Auto-Correlation Function

We have PSD $S_{xx}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$

$$= \lim_{T\to\infty} \frac{E[X^*{}_T(\omega)X_T(\omega).]}{2T}$$

$X_T(\omega) = \int_{-T}^{T} X(t).e^{-j\omega t}.dt$ and $X^*{}_T(\omega) = \int_{-T}^{T} X(t).e^{j\omega t}.dt$

$$S_{xx}(\omega) = \lim_{T\to\infty} \frac{1}{2T} E\left[\int_{-T}^{T} X(t_1).e^{j\omega t_1}.dt_1. \int_{-T}^{T} X(t_2).e^{-j\omega t_2}.dt_2\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} E\left[\int_{-T}^{T}\int_{-T}^{T} X(t_1)\,X(t_2).e^{-j\omega(t_2-t_1)}.dt_1dt_2.\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} E[X(t_1)\,X(t_2)].e^{-j\omega(t_2-t_1)}.dt_1dt_2$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} R_{xx}(t_1,t_2)\,e^{-j\omega(t_2-t_1)}.dt_1dt_2$$

Consider the inverse Fourier Transform of PSD i.e. $\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)\,e^{j\omega\tau}.d\omega$

$F^{-1}[S_{xx}(\omega)] =$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T} R_{xx}(t_1,t_2)\,e^{-j\omega\,(t_2-t_1)}.dt_1dt_2]\,e^{j\omega\tau}.d\omega$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T} R_{xx}(t_1,t_2)\,\frac{1}{2\pi}.\int_{-\infty}^{\infty} e^{-j\omega\,(t_2-t_1)}.d\omega.dt_1dt_2$$

Since, $F[\delta(t)] = 1, \to \frac{1}{2\pi}\int_{-\infty}^{\infty} 1.e^{j\omega t}\,d\omega = \delta(t)$

On similar lines, $\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega\,(\tau-t_2+t_1)}\,d\omega = \delta(\tau - t_2 + t_1)$

$F^{-1}[S_{xx}(\omega)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \int_{-T}^{T} R_{xx}(t_1, t_2)\delta(\tau - t_2 + t_1).\, dt_1 dt_2$

since $\delta(\tau - t_2 + t_1) = 1\ at\ \tau - t_2 + t_1 = 0\ i.e. t_2 = \tau + t_1$

$F^{-1}[S_{xx}(\omega)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} R_{xx}(t_1, \tau + t_1)\, dt_1$

Let $t_1 = \tau \rightarrow dt_1 = d\tau$

Hence, $F^{-1}[S_{xx}(\omega)] = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} R_{xx}(t, t+\tau)\, dt$

The RHS of the above eq. is the time average of Auto correlation function.
Thus, Time average of Autocorrelation function and the PSD form a Fourier Transform Pair.

4. Power Density Spectrum of Response

Using the property of Fourier transform, we get the power spectral density of the output process given by

$$\begin{aligned} S_Y(\omega) &= S_X(\omega) H(\omega) H^*(\omega) \\ &= S_X(\omega) |H(\omega)|^2 \end{aligned}$$

Also note that

$$\begin{aligned} R_{XY}(\tau) &= h(-\tau) * R_X(\tau) \\ and\ R_{YX}(\tau) &= h(\tau) * R_X(\tau) \end{aligned}$$

Taking the Fourier transform of $R_{XY}(\tau)$ we get the cross power spectral density $S_{XY}(\omega)$ given by

$$S_{XY}(\omega) = H^*(\omega) S_X(\omega)$$

and

$$S_{YX}(\omega) = H(\omega) S_X(\omega)$$

5. Cross-Power Density Spectrums of Input and Output

The Cross correlation of the input {X*(t)*} and the out put *{Y (t)}* is given by

$$\begin{aligned} E\left(X(t+\tau)Y(t)\right) &= E\ X(t+\tau) \int_{-\infty}^{\infty} h(s)\ X(t-s)\ ds \\ &= \int_{-\infty}^{\infty} h(s)\quad E\ X(t+\tau)\ X(t-s)\ ds \\ &= \int_{-\infty}^{\infty} h(s)\quad R_X(\tau+s) ds \\ &= \int_{-\infty}^{\infty} h(-u)\quad R_X(\tau-u) du \qquad [\text{ Put } s = -u\ ] \\ &= h(-\tau) * R_X(\tau) \end{aligned}$$

$$\begin{aligned} \therefore\quad R_{XY}(\tau) &= h(-\tau) * R_X(\tau) \\ also\quad R_{YX}(\tau) &= R_{XY}(-\tau) = h(\tau) * R_X(-\tau) \\ &= h(\tau) * R_X(\tau) \end{aligned}$$

14. Tutorial Topics and Questions

- Orthogonal signal space
- Linear time invariant systems
- Graphical representation of convolution
- Fourier transform of standard signals
- Sampling theorem
- Relation between L.T and F.T of a signal
- Properties of z-transforms
- Distribution and density functions
- Time averages and ergodicity

- Auto correlation function of response
- Relationship between power spectrum and auto correlation function
- Power density spectrum of response

Questions

A. Explain the orthogonal signal space

B. Explain the linear time invariant systems

C. Draw and explain the graphical representation of convolution

D. Determine the Fourier transform of standard signals

E. State and prove the sampling theorem

F. Derive the relation between L.T and F.T of a signal

G. State and prove any four properties of z-transforms

H. Explain distribution and density functions

I. Write short notes on time averages and ergodicity

J. Explain the response of auto correlation function

K. Derive the Relationship between power spectrum and auto correlation function

L. Explain the response of power density spectrum

15) UNIT WISE-QUESTION BANK

UNIT-I

2 MARKS QUESTIONS WITH ANSWERS

1. **Define Signal**.

Ans: A signal is a function of one or more independent variables which contain some information. Eg: Radio signal, TV signal, Telephone signal etc.

2. Define System.

Ans: A system is a set of elements or functional block that are connected together and produces an output in response to an input signal.

Eg: An audio amplifier, attenuator, TV set etc.

3. Define unit step, ramp and delta functions for CT.

Ans: Unit step function is defined as

U(t) = 1 for t >= 0

0 otherwise

Unit ramp function is defined as

r(t) = t for t>=0

0 for t<0

Unit delta function is defined as

δ(t)= 1 for t=0

4. Define linear and non-linear systems.

Ans: A system is said to be linear if superposition theorem applies to that system. If it does not satisfy the superposition theorem, then it is said to be a nonlinear system.

5. Define Causal and non-Causal systems.

Ans: A system is said to be a causal if its output at anytime depends upon present and past inputs only. A system is said to be non-causal system if its output depends upon future inputs also.

3 MARKS QUESTIONS WITH ANSWERS

1. State the classification or characteristics of CT and DT systems.

Ans: The DT and CT systems are according to their characteristics as follows

(i). Linear and Non-Linear systems

(ii). Time invariant and Time varying systems.

(iii). Causal and Non causal systems.

(iv). Stable and unstable systems.

(v). Static and dynamic systems.

(vi). Inverse systems.

2. Discuss the properties of Convolution?

Ans: Commutative Property

$x1\ (t) * x2\ (t) = x2\ (t) * x1\ (t)$

Distributive Property

$x1\ (t) * [x2\ (t) + x3\ (t)] = [x1\ (t) * x2\ (t)] + [x1\ (t) * x3\ (t)]$

Associative Property

$x1\ (t) * [x2\ (t) * x3\ (t)] = [x1\ (t) * x2\ (t)] * x3\ (t)$

Shifting Property

$x1\ (t) * x2\ (t) = y(t)$

$x1\ (t) * x2\ (t - t0\ ) = y(t - t0\ )\ x1\ (t - t0\ ) * x2\ (t) = y(t - t0\ )$

$x1\ (t - t0\ ) * x2\ (t - t1\ ) = y(t - t0 - t1\ )$

Convolution with Impulse

$x1\ (t) * \delta(t) = x(t)$

$x1\ (t) * \delta(t - t0\ ) = x(t - t0\ )$

Convolution of Unit Steps

$u(t) * u(t) = r(t)$

$u(t - T1) * u(t - T2) = r(t - T1 - T2)$

$u(n) * u(n) = [n + 1]u(n)$

Scaling Property

If $x(t) * h(t) = y(t)$

then $x(at) * h(at) = \frac{1}{|a|} y(at)$

Differentiation of Output

if $y(t) = x(t) * h(t)$

then $\frac{dy(t)}{dt} = \frac{dx(t)}{dt} * h(t)$

or

$\frac{dy(t)}{dt} = x(t) * \frac{dh(t)}{dt}$

3. What is Closed and Complete Set of Orthogonal Functions

Ans: Let us consider a set of n mu(t)ually orthogonal functions $x_1 t$, $x_2 t ... x_n t$ over the interval t_1 to t_2. This is called as closed and complete set when there exist no function f t satisfying the condition

$\int_{t_1}^{t_2} f(t)x_k(t)dt = 0$

If this function is satisfying the equation $\int_{t_1}^{t_2} f(t)x_k(t)dt = 0$ for $k = 1, 2, ..$ then ft is said to be orthogonal to each and every function of orthogonal set. This set is incomplete withou(t) ft. It becomes closed and complete set when f t is included.

ft can be approximated with this orthogonal set by adding the components along mutually orthogonal signals i.e.

$$f(t) = C_1 x_1(t) + C_2 x_2(t) + ... + C_n x_n(t) + f_e(t)$$

If the infinite series $C_1 x_1(t) + C_2 x_2(t) + ... + C_n x_n(t)$ converges to ft then mean square error is zero.

4. Write short on Mean Square Error

Ans: The average of square of error function f_e t is called as mean square error. It is denoted by ε *epsilon* .

$$\varepsilon = \frac{1}{t_2-t_1}\int_{t_1}^{t_2}[f_e(t)]^2dt$$

$$= \frac{1}{t_2-t_1}\int_{t_1}^{t_2}[f_e(t) - \Sigma_{r=1}^{n} C_r x_r(t)]^2dt$$

$$= \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f_e^2(t)]dt + \Sigma_{r=1}^{n} C_r^2 \int_{t_1}^{t_2} x_r^2(t)dt - 2\Sigma_{r=1}^{n} C_r \int_{t_1}^{t_2} x_r(t)f(t)dt$$

You know that $C_r^2 \int_{t_1}^{t_2} x_r^2(t)dt = C_r \int_{t_1}^{t_2} x_r(t)f(d)dt = C_r^2 K_r$

$$\varepsilon = \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f^2(t)]dt + \Sigma_{r=1}^{n} C_r^2 K_r - 2\Sigma_{r=1}^{n} C_r^2 K_r]$$

$$= \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f^2(t)]dt - \Sigma_{r=1}^{n} C_r^2 K_r]$$

$$\therefore \varepsilon = \frac{1}{t_2-t_1}[\int_{t_1}^{t_2}[f^2(t)]dt + (C_1^2 K_1 + C_2^2 K_2 + \ldots + C_n^2 K_n)]$$

The above equation is used to evaluate the mean square error.

5. Determine whether the signal x (t) described by x (t) = exp [- at] u (t), a > 0 is a power signal or energy signal or neither.

Ans:

$$x(t) = e^{-at} u(t),\ a > 0$$

x(t) is a non-periodic signal.

$$\text{Energy } E = \int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{0}^{\infty} e^{-2at}\,dt = \left.\frac{e^{-2at}}{-2a}\right|_{0}^{\infty} = \frac{1}{2a} \quad \text{(finite, positive)}$$

The energy is finite and deterministic.

$\therefore$ x(t) is an energy signal.

5 MARKS QUESTIONS WITH ANSWERS

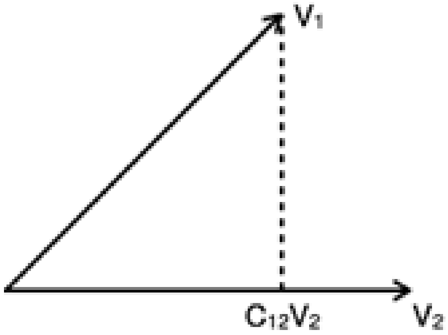
1. What is the difference between the Vectors and Signals

Ans: There is a perfect analogy between vectors and signals.

Vector

A vector contains magnitude and direction. The name of the vector is denoted by bold face type and their magnitude is denoted by light face type.

Example: V is a vector with magnitude V. Consider two vectors V_1 and V_2 as shown in the following diagram. Let the component of V_1 along with V_2 is given by $C_{12}V_2$. The component of a vector V_1 along with the vector V_2 can obtained by taking a perpendicular from the end of V_1 to the vector V_2 as shown in diagram



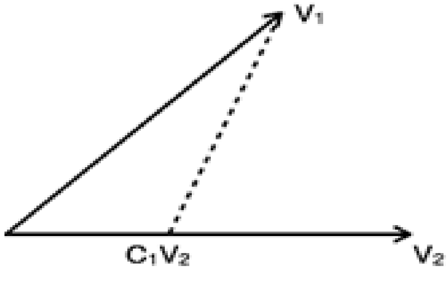
The vector V_1 can be expressed in terms of vector V_2

$$V_1 = C_{12}V_2 + V_e$$

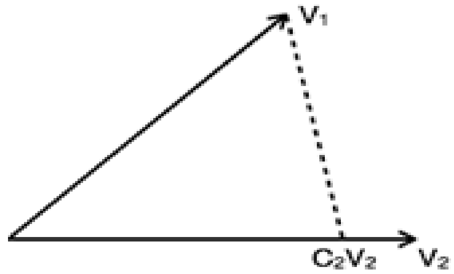
Where Ve is the error vector.

Bu(t) this is not the only way of expressing vector V_1 in terms of V_2. The alternate possibilities are:

$V_1 = C_1V_2 + V_{e1}$



$V_2 = C_2 V_2 + V_{e2}$



The error signal is minimum for large component value. If $C_{12}=0$, then two signals are said to be orthogonal.

Dot Product of Two Vectors

$V_1 \cdot V_2 = V_1.V_2 \cos\theta$

θ = Angle between V1 and V2

$V_1 \cdot V_2 = V_2.V_1$

The components of V_1 along $V_2 = V_1 \cos\theta = \frac{V1.V2}{V2}$

From the diagram, components of V_1 along $V_2 = C_{12} V_2$

$$\Rightarrow C_{12} = \frac{V_1 \cdot V_2}{V^2}$$

Signal

The concept of orthogonality can be applied to signals. Let us consider two signals $f_1(t)$and $f_2(t)$. Similar to vectors, you can approximate $f_1(t)$in terms of f2(t)

Vectors and Signals = C_{12}

$f2(t) + f_e t$ for $(t_1 < t < t_2) \Rightarrow f_e t =$

$f1(t) - C_{12}\, f2(t)$

One possible way of minimizing the error is integrating over the interval t_1 to t_2.

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [f_e(t)]dt$$

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)]dt$$

However, this step also does not reduce the error to appreciable extent. This can be corrected by taking the square of error function.

$$\varepsilon = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t)]^2 dt$$

$$\Rightarrow \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_e(t) - C_{12} f_2]^2 dt$$

Where ε is the mean square value of error signal. The value of C_{12} which minimizes the error, you need to calculate $\frac{d\varepsilon}{dC_{12}} = 0$

$$\Rightarrow \frac{d}{dC_{12}} \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} [f_1(t) - C_{12} f_2(t)]^2 dt\right] = 0$$

$$\Rightarrow \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left[\frac{d}{dC_{12}} f_1^2(t) - \frac{d}{dC_{12}} 2 f_1(t) C_{12} f_2(t) + \frac{d}{dC_{12}} f_2^2(t) C_{12}^2\right] dt = 0$$

Derivative of the terms which do not have C12 term are zero.

$$\Rightarrow \int_{t_1}^{t_2} -2 f_1(t) f_2(t) dt + 2 C_{12} \int_{t_1}^{t_2} [f_2^2(t)] dt = 0$$

If $C_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) dt}{\int_{t_1}^{t_2} f_2^2(t) dt}$ component is zero, then two signals are said to be orthogonal.

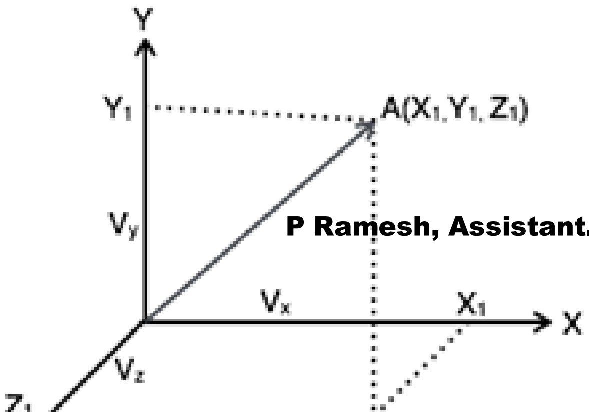
Put $C_{12} = 0$ to get condition for orthogonality.

$$0 = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) dt}{\int_{t_1}^{t_2} f_2^2(t) dt}$$

$$\int_{t_1}^{t_2} f_1(t) f_2(t) dt = 0$$

2. Explain Orthogonal Vector Space?

Ans: A complete set of orthogonal vectors is referred to as orthogonal vector space. Consider a three dimensional vector space as shown below:



Consider a vector A at a point (X_1, Y_1, Z_1). Consider three unit vectors (V_X, V_Y, V_Z) in the direction of X, Y, Z axis respectively. Since these unit vectors are mu(t)ually orthogonal, it satisfies that

$$V_X . V_X = V_Y . V_Y = V_Z . V_Z = 1$$
$$V_X . V_Y = V_Y . V_Z = V_Z . V_X = 0$$

You can write above conditions as $V_a . V_b = \begin{cases} 1, & a = b \\ 0, & a \neq b \end{cases}$

The vector A can be represented in terms of its components and unit vectors as

$A = X_1 V_X + Y_1 V_Y + Z_1 V_Z \dots\dots\dots\dots\dots\dots (1)$

Any vectors in this three dimensional space can be represented in terms of these three unit vectors only.

If you consider n dimensional space, then any vector A in that space can be represented as

$A = X_1 V_X + Y_1 V_Y + Z_1 V_Z + \dots + N_1 V_N \dots\dots (2)$

As the magnitude of unit vectors is unity for any vector A

The component of A along x axis = A.V_X
The component of A along Y axis = A.V
The component of A along Z axis = A.V_Z

Similarly, for n dimensional space, the component of A along some G axis

$= A. VG \dots\dots\dots\dots\dots\dots (3)$

Substitu(t)e equation 2 in equation 3.

$\Rightarrow CG = (X_1 V_X + Y_1 V_Y + Z_1 V_Z + \ldots + G_1 V_G \ldots + N_1 V_N) V_G$

$= X_1 V_X V_G + Y_1 V_Y V_G + Z_1 V_Z V_G + \ldots + G_1 V_G V_G \ldots + N_1 V_N V_G = G_1$

since $V_G V_G = 1$

$If V_G V_G \neq 1$ i.e. $V_G V_G = k\, AV$

$G = G_1 V_G V_G = G_1 K$

$$G_1 = \frac{(AV_G)}{K}$$

3. Explain Orthogonal Signal Space?

Ans: Let us consider a set of n mu(t)ually orthogonal functions $x_1 t$, $x_2 t$... $x_n t$ over the interval t_1 to t_2. As these functions are orthogonal to each other, any two signals $x_j(t)$, $x_k(t)$ have to satisfy the orthogonality condition. i.e.

$$\int_{t_1}^{t_2} x_j(t) x_k(t) dt = 0 \;\; \text{where}\, j \neq k$$

$$\text{Let} \int_{t_1}^{t_2} x_k^2(t) dt = k_k$$

Let a function ft, it can be approximated with this orthogonal signal space by adding the components along mu(t)ually orthogonal signals i.e.

$$f(t) = C_1 x_1(t) + C_2 x_2(t) + \ldots + C_n x_n(t) + f_e(t)$$
$$= \Sigma_{r=1}^{n} C_r x_r(t)$$
$$f(t) = f(t) - \Sigma_{r=1}^{n} C_r x_r(t)$$

Mean sqaure error

$$\varepsilon = \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f_e(t)]^2 dt$$
$$= \frac{1}{t_2 - t_2} \int_{t_1}^{t_2} [f[t] - \sum_{r=1}^{n} C_r x_r(t)]^2 dt$$

The component which minimizes the mean square error can be found by

$$\frac{d\varepsilon}{dC_1} = \frac{d\varepsilon}{dC_2} = \ldots = \frac{d\varepsilon}{dC_k} = 0$$

Let us consider $\frac{d\varepsilon}{dC_k} = 0$

$$\frac{d}{dC_k}[\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} [f(t) - \Sigma_{r=1}^{n} C_r x_r(t)]^2 dt] = 0$$

All terms that do not contain C_k is zero. i.e. in summation, r=k term remains and all other terms are zero.

$$\int_{t_1}^{t_2} -2f(t)x_k(t)dt + 2C_k \int_{t_1}^{t_2} [x_k^2(t)]dt = 0$$

$$\Rightarrow C_k = \frac{\int_{t_1}^{t_2} f(t)x_k(t)dt}{int_{t_1}^{t_2} x_k^2(t)dt}$$

$$\Rightarrow \int_{t_1}^{t_2} f(t)x_k(t)dt = C_k K_k$$

e) Discuss in detail the classifications of systems?

Ans: Systems are classified into the following categories:

- Liner and Non-liner Systems
- Time Variant and Time Invariant Systems
- Liner Time variant and Liner Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Invertible and Non-Invertible Systems
- Stable and Unstable Systems

Liner and Non-liner Systems

A system is said to be linear when it satisfies superposition and homogenate principles. Consider two systems with inputs as x1(t) , x2(t) , and outputs as y1(t) , y2(t) respectively. Then, according to the superposition and homogenate principles,

T [a1 x1(t) + a2 x2(t)] = a1 T[x1(t)] + a2 T[x2(t)] ∴, T [a1 x1(t) + a2 x2(t)] = a1 y1(t) + a2 y2(t)

From the above expression, is clear that response of overall system is equal to response of individual system.

Example:

t = x2(t)

Solution:

y1 (t)= T[x1(t)] = x12(t)

y2 (t)= T[x2(t)] = x22(t)

T [a1 x1(t) + a2 x2(t)] = [a1 x1(t) + a2 x2(t)]2

Which is not equal to a1 y1(t) + a2 y2(t). Hence the system is said to be non linear.

Time Variant and Time Invariant Systems

A system is said to be time variant if its input and output characteristics vary with time. Otherwise, the system is considered as time invariant.

The condition for time invariant system is:
$x(t-t_0) = y(t-t_0)$

The condition for time variant system is:

$x(t-t_0) \neq y(t-t_0)$

Example:
y(t)=2x(t)
Liner Time variant *LTV* **and Liner Time Invariant** *LTI* **Systems**

If a system is both liner and time variant, then it is called liner time variant *LTV* system.

If a system is both liner and time Invariant then that system is called liner time invariant *LTI* system.

Static and Dynamic Systems

Static system is memory-less whereas dynamic system is a memory system.

Example 1: y(t) = 2 x(t)

For present value t=0, the system output is y0 = 2x0. Here, the output is only dependent upon present input. Hence the system is memory less or static.

Example 2: y(t) = 2 x(t) + 3 x(t)− 3

For present value t=0, the system output is y0 = 2x0 + 3x−3.

Here x−3 is past value for the present input for which the system requires memory(t)o get this output.

Hence, the system is a dynamic system.

Causal and Non-Causal Systems

A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input.

For non causal system, the output depends upon future inputs also.

Example 1: $yn = 2\ x(t) + 3\ x(t) - 3$

For present value t=1, the system output is $y1 = 2x1 + 3x-2$.

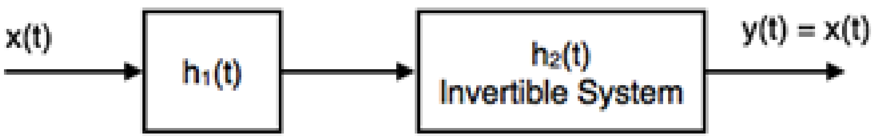
Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Example 2: $yn = 2\ x(t) + 3\ x(t) - 3 + 6x(t) + 3$

For present value t=1, the system output is $y1 = 2x1 + 3x-2 + 6x4$ Here, the system output depends upon future input. Hence the system is non-causal system.

Invertible and Non-Invertible systems

A system is said to invertible if the input of the system appears at the output.



$YS = XS\ H1S\ H2S$

$= XS\ H1S \cdot (H1(S)) \quad Since\ H2S = 1/H1(S)$

$\therefore, YS = XS$
$\rightarrow y(t) = x(t)$

Hence, the system is invertible.

If $y(t) \neq x(t)$, then the system is said to be non-invertible.

Stable and Unstable Systems

The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system then it is said to be unstable.

Note: For a bounded signal, amplitude is finite.

Example 1: y(t) = x2(t)

Let the input is u*t* *unit step bounded input* then the output y(t) = u2*t* = u*t* = bounded output.

Hence, the system is stable.

Example 2: y(t) = ∫ *x*(*t*)*dt*

Let the input is u *t unit step bounded input* then the output y(t) = ∫ *u*(*t*)*dt* = ramp signal *unbounded because amplitude of ramp is not finite it goes to infinite when t → infinite* .Hence, the system is unstable.

5. Define the convolution? And explain the procedure to determine the response of the LTI system?

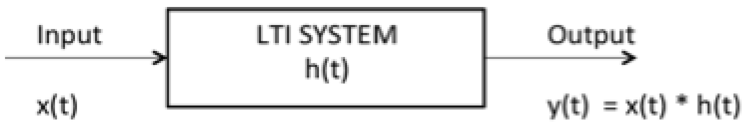
Ans: **Convolution**

Convolution is a mathematical operation used to express the relation between input and output of an LTI system. It relates input, output and impulse response of an LTI system as
$y(t) = x(t) * h(t)$

Where y *t* = output of LTI
x *t* = input of LTI
h *t* = impulse response of LTI There are two types of convolutions:

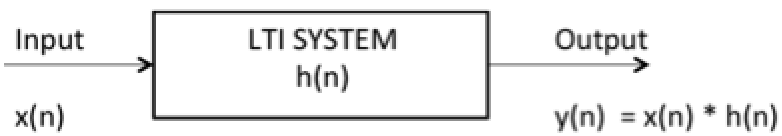
- Continuous convolution
- Discrete convolution

Continuous Convolution



$$
\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&\quad or \\
&= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau
\end{aligned}
$$

Discrete Convolution



$y(n) = x(n) * h(n)$

$$= \Sigma_{k=-\infty}^{\infty} x(k)h(n-k)$$

or

$$= \Sigma_{k=-\infty}^{\infty} x(n-k)h(k)$$

By using convolution we can find zero state response of the system.

Deconvolution

Deconvolution is reverse process to convolution widely used in signal and image processing.

Multiple Choice Questions

1. What is the order of the four operations that are needed to be done on h(k) in order to convolute x(k) and h(k)?

Step-1: Folding

Step-2: Multiplication with x(k)

Step-3: Shifting

Step-4: Summation

a) 1-2-3-4 b) 1-2-4-3

c) 2-1-3-4 d) 1-3-2-4

2. The impulse response of a LTI system is h(n)={1,1,1}. What is the response of the signal to the input x(n)={1,2,3}?

a) {1,3,6,3,1} b) {1,2,3,2,1}

c) {1,3,6,5,3} d) {1,1,1,0,0}

3. x(n)*(h1(n)*h2(n))=(x(n)*h1(n))*h2(n)

a) True

b) False

4. Determine the impulse response for the cascade of two LTI systems having impulse responses h1(n)=(1/2) u(n) and h2(n)= (1/4) u(n).

a) (1/2) [2-(1/2)], n<0 b) (1/2) [2-(1/2)], n>0

c) (1/2) [2+(1/2)], n<0 d) (1/2) [2+(1/2)], n>0

5. An LTI system is said to be causal if and only if

a) Impulse response is non-zero for positive values of n

b) Impulse response is zero for positive values of n

c) Impulse response is non-zero for negative values of n

d) Impulse response is zero for negative values of n

6. Is the system with impulse response h(n)=2 u(n-1) stable?

a) True

b) False

7. Two signals are said to be orthogonal if the angle between the signals is______ degrees

a. 90 b. 30

c. 45 d. 180

8. _____filter cannot be realized

a. band stop b. band pass

c. all pass d. Ideal

9. Impulse response of the LTI system is give by

a. Output of the LTI system when the input is impulse function

b. Output of the LTI system when the input is unit step function

c. Output of the LTI system when the input is unit ramp function

d. Output of the LTI system when the input is unit parabolic function

10. Paley Wiener criterion for designing of filter is

a. $\int_{-\infty}^{+\infty} \frac{|\mathrm{In}\ (H(\omega))|}{1+\omega^2} d\omega < \infty$

b. $\int_{-\infty}^{+\infty} \frac{|\log(H(\omega))|}{1+\omega^2} d\omega < \infty$

c. $\int_{-\infty}^{+\infty} \left| H(\omega)^2 \right| d\omega < \infty$

d. a and b

Key

1. d
2. c
3. a
4. b
5. d
6. b
7. a
8. d
9. a
10. a

FILL IN THE BLANKS

1. Time reflection is also known as the time _____

2. __________ is an entity that manipulates on one or more signals and produces a new signal
3. A useful property of the unit impulse $\delta(t)$ is _____

4. The response of a linear time invariant system for the input x(t) and the impulse response h(t) is ____________

5. A given system is characterized by the differential equation
$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t).$$ The system is:

6. The system characterized by the equation y(t) = ax(t)+ b is________
7. The continuous time system described by y (t) = x $(t)^2$ is____
8. The system having input x(n) related to output y(n) as y(n) = log10 x(n) is:_____
9. The system represented by y(n) = x(-n) is _____

10. If $\int x_1(t)x_2(t)\, dt = 0$ then the two signals are said to be___________

KEY:

1	Time folding
2	System
3	$\delta(at) = \frac{1}{a}\delta(t),\ a > 0$
4	y(t)=x(t)*h(t)
5	linear and unstable
6	non-linear.
7	non causal, linear and time-invariant
8	nonlinear, causal, stable.
9	Linear
10	orthogonal signals

UNIT-II

2 MARKS QUESTIONS WITH ANSWERS

1. Define Sampling

Ans: Sampling is a process of converting a continuous time signal into discrete time Signal. After sampling the signal is defined at discrete instants of time and the time Interval between two subsequent sampling instants is called sampling interval.

2.Define Fourier series

Ans: To represent any periodic signal x*t*, Fourier developed an expression called Fourier series. This is in terms of an infinite sum of sines and cosines or exponentials. Fourier series uses orthoganality condition.

3. Define Fourier transform pair

Ans: For every time domain waveform there is a corresponding frequency domain waveform, and vice versa. For example, a rectangular pulse in the time domain coincides with a sinc function in the frequency domain. Duality provides that the reverse is also true; a rectangular pulse in the frequency domain matches a sinc function in the time domain. Waveforms that correspond to each other in this manner are called Fourier transform pairs

4. State linearity property of Fourier transform

Ans: Linearity Property

If$x(t) \overset{F.T}{\longleftrightarrow} X(\omega)Ifx(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

&$y(t) \overset{F.T}{\longleftrightarrow} Y(\omega)$&$y(t) \overset{F.T}{\longleftrightarrow} Y(\omega)$

Then linearity property states that

ax(t)+by(t)⟷F.TaX(ω)+bY(ω)ax(t)+by(t)⟷F.TaX(ω)+bY(ω)

5. Define Nyquist rate and Nyquist interval.

Ans: **Nyquist rate:** When the sampling rate becomes exactly equal to 2W samples/sec, for a given bandwidth of fm or W Hertz, then it is called as Nyquist rate.

Nyquist rate = $2f_m$ samples/second

Nyquist interval: It is the time interval between any two adjacent samples when sampling rate is Nyquist rate.

Nyquist interval = 1/2W or $1/2f_m$

3 MARKS QUESTIONS WITH ANSWERS

1. Write the Conditions for Existence of Fourier Transform

Ans: Any function f(*t*) can be represented by using Fourier transform only when the function satisfies Dirichlet's conditions. i.e.

- The function f(*t*)has finite number of maxima and minima.
- There must be finite number of discontinuities in the signal f*t*,in the given interval of time.
- It must be absolutely integrable in the given interval of time i.e.

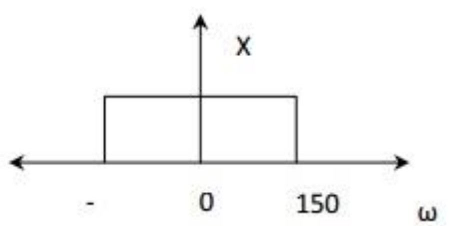
$$\int^{\infty} |f(t)|dt < \infty$$

2. A signal x(t) = sinc (150πt) is sampled at a rate of a) 100 Hz, b) 200 Hz, and c) 300Hz. For each of these cases, explain if you can recover the signal x(t) from the sampled signal.

Solution:

Given x(t) = sinc (150πt)

The spectrum of the signal x(t) is a rectangular pulse with a bandwidth (maximum frequency component) of 150π rad/sec as shown in figure.



$2\pi f_m = 150\pi$

$f_m = 75$ Hz

Nyquist rate is $2f_m = 150$ Hz

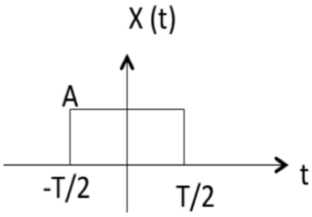
· For the first case the sampling rate is 100Hz, which is less than Nyquist rate (under sampling). Therefore x(t) cannot be recovered from its samples.

· And (c) in both cases the sampling rate is greater than Nyquist rate. Therefore x(t) can be recovered from its sample.

3. Determine the Fourier Transform of Gate and impulse Functions?

Sol:

FT of GATE Function



$$F[\omega] = ATSa(\frac{\omega T}{2})$$

FT of Impulse Function

$FT[\omega(t)] = [\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt] = e^{-j\omega t}|_{t=0}$

$= e^0 = 1$

$\therefore \delta(\omega) = 1$

4. List any four properties of Fourier Transform?

Ans: Here are the properties of Fourier Transform:

1) Linearity Property

If $x(t) \xleftrightarrow{F.T} X(\omega)$

& $y(t) \xleftrightarrow{F.T} Y(\omega)$

Then linearity property states that

$ax(t) + by(t) \xleftrightarrow{F.T} aX(\omega) + bY(\omega)$

2) Time Shifting Property

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

Then Time shifting property states that

$x(t - t_0) \overset{F.T}{\longleftrightarrow} e^{-j\omega t_0} X(\omega)$

3) Frequency Shifting Property

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

Then frequency shifting property states that

$e^{j\omega_0 t}.x(t) \overset{F.T}{\longleftrightarrow} X(\omega - \omega_0)$

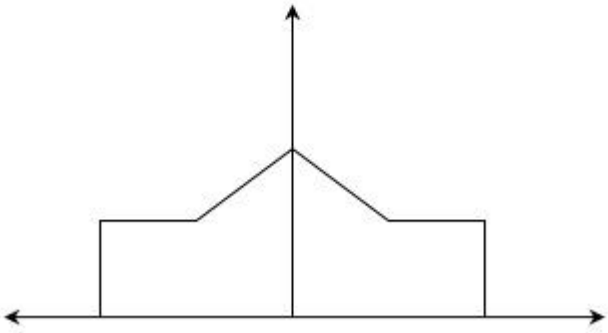
4) Time Reversal Property

If $x(t) \overset{F.T}{\longleftrightarrow} X(\omega)$

Then Time reversal property states that

$x(-t) \overset{F.T}{\longleftrightarrow} X(-\omega)$

5.A signal x(t) whose spectrum is shown in figure is sampled at a rate of 300 samples/sec. What is the spectrum of the sampled discrete time signal.



Solution:

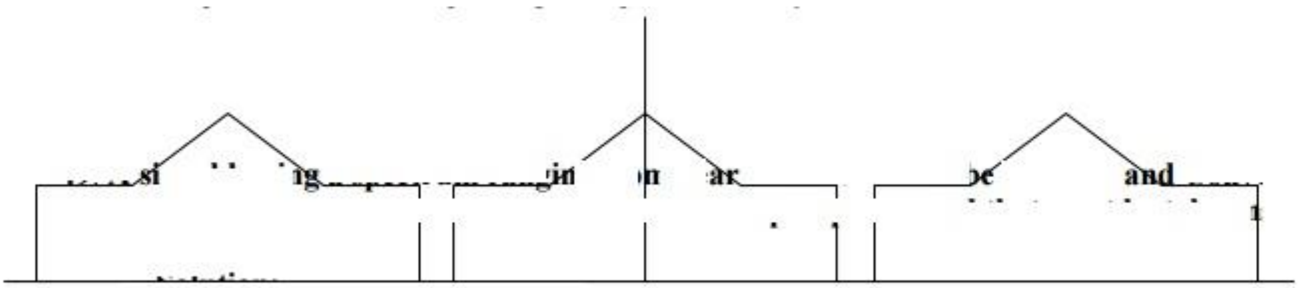
f_m = 100 Hz

Nyquist rate = $2f_m$ = 200 Hz

Sampling frequency = f_s = 300 Hz

$f_s > 2f_m$, Therefore no aliasing takes place

The spectrum of the sampled signal repeats for every 300 Hz.



5 MARKS QUESTIONS WITH ANSWERS

1. **Explain about the Fourier Series Representation of Continuous Time Periodic Signals**

Ans**: Fourier series**

To represent any periodic signal xt, Fourier developed an expression called Fourier series. This is in terms of an infinite sum of sines and cosines or exponentials. Fourier series uses orthoganality condition.

Fourier Series Representation of Continuous Time Periodic Signals

A signal is said to be periodic if it satisfies the condition x t = x $t + T$ or x n = x $n + N$.

Where T = fundamental time period,

ω_0= fundamental frequency = $2\pi/T$

There are two basic periodic signals:

$x(t) = \cos \omega_0 t$ *sinusoidal* &

$x(t) = e^{j\omega 0t}$ *complex exponential*

These two signals are periodic with period $T = 2\pi/\omega_0$.

A set of harmonically related complex exponentials can be represented as $\{\phi_k(t)\}$

$$\phi_k(t) = \{e^{jk\omega_0 t}\} = \{e^{jk(\frac{\overline{\frac{T}{2\pi}}}{T})t}\} \text{where } k = 0 \pm 1, \pm 2..n \; .....(1)$$

All these signals are periodic with period T

According to orthogonal signal space approximation of a function x t with n, mutually orthogonal functions is given by

$$x(t) = \sum^{\infty} a_k e^{jk\omega_0 t} .....(2)$$

$$\sum_{k=-\infty} a_k k e^{jk\omega_0 t}$$

Where a_k= Fourier coefficient = coefficient of

approximation. This signal xt is also periodic with period T.

Equation 2 represents Fourier series representation of periodic signal xt.

The term k = 0 is constant.

The term $k = \pm 1$ having fundamental frequency ω_0, is called as 1st harmonics.

The term $k = \pm 2$ having fundamental frequency $2\omega_0$, is called as 2nd harmonics, and so on...

The term $k = \pm n$ having fundamental frequency $n\omega 0$, is called as nth harmonics.

Deriving Fourier Coefficient

We know that $x(t) = \Sigma^{\infty}{}_{k=\infty} a_k e^{jk\omega 0t} ......(1)$

Multiply $e^{-jn\omega 0t}$ on both sides. Then 0

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.e^{-jn\omega_0 t}$$

Consider integral on both sides.

We know that $x(t) = \Sigma_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \ldots\ldots(1)$

Multiply $e^{-jn\omega_0 t}$ on both sides. Then

$$x(t)e^{-jn\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.e^{-jn\omega_0 t}$$

Consider integral on both sides.

$$\int_0^T x(t)e^{jk\omega_0 t}dt = \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.e^{-jn\omega_0 t}dt$$

$$= \int_0^T \sum_{k=-\infty}^{\infty} a_k e^{j(k-n)\omega_0 t}.dt$$

$$\int_0^T x(t)e^{jk\omega_0 t}dt = \sum_{k=-\infty}^{\infty} a_k \int_0^T e^{j(k-n)\omega_0 t}\,dt.\ \ldots\ldots(2)$$

by Euler's formula,

$$\int_0^T e^{j(k-n)\omega_0 t}dt. = \int_0^T \cos(k-n)\omega_0 dt + j\int_0^T \sin(k-n)\omega_0 t\,dt$$

$$\int_0^T e^{j(k-n)\omega_0 t}\,dt. = \begin{cases} T & k = n \\ 0 & k \neq n \end{cases}$$

Hence in equation 2, the integral is zero for all values of k except at k = n. Put k = n in equation 2.

Hence in equation 2, the integral is zero for all values of k except at k = n. Put k = n in equation 2.

2. Determine the relation Between Trigonometric and Exponential Fourier Series?

Ans**: Trigonometric Fourier Series** *TFS*

$\sin n\omega_0 t$ and $\sin m\omega_0 t$ are orthogonal over the interval $(t_0, t_0 + \frac{2\pi}{\omega_0})$. So $\sin \omega_0 t$, $\sin 2\omega_0 t$ forms an orthogonal set. This set is not complete without $\{\cos n\omega_0 t\}$ because this cosine set is also orthogonal to sine set. So to complete this set we must include both cosine and sine terms. Now the complete orthogonal set contains all cosine and sine terms i.e. $\{\sin n\omega_0 t, \cos n\omega_0 t\}$ where

n=0, 1, 2...

$\therefore$ Any function xt in the interval $(t_0, t_0 + \frac{2\pi}{\omega_0})$ can be represented as

$$x(t) = a_0 \cos 0\omega_0 t + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \ldots + a_n \cos n\omega_0 t + \ldots$$
$$+ b_0 \sin 0\omega_0 t + b_1 \sin 1\omega_0 t + \ldots + b_n \sin n\omega_0 t + \ldots$$
$$= a_0 + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \ldots + a_n \cos n\omega_0 t + \ldots$$
$$+ b_1 \sin 1\omega_0 t + \ldots + b_n \sin n\omega_0 t + \ldots$$

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \quad (t_0 < t < t_0 + T)$$

The above equation represents trigonometric Fourier series representation of xt.

$$\text{Where } a_0 = \frac{\int_{t_0}^{t_0+T} x(t) \cdot 1 dt}{\int_{t_0}^{t_0+T} 1^2 dt} = \frac{1}{T} \cdot \int_{t_0}^{t_0+T} x(t) dt$$

$$a_n = \frac{\int_{t_0}^{t_0+T} x(t) \cdot \cos n\omega_0 t \, dt}{\int_{t_0}^{t_0+T} \cos^2 n\omega_0 t \, dt}$$

$$b_n = \frac{\int_{t_0}^{t_0+T} x(t) \cdot \sin n\omega_0 t \, dt}{\int_{t_0}^{t_0+T} \sin^2 n\omega_0 t \, dt}$$

$$\text{Here } \int_{t_0}^{t_0+T} \cos^2 n\omega_0 t \, dt = \int_{t_0}^{t_0+T} \sin^2 n\omega_0 t \, dt = \frac{T}{2}$$

$$\therefore a_n = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \cos n\omega_0 t \, dt$$

$$b_n = \frac{2}{T} \cdot \int_{t_0}^{t_0+T} x(t) \cdot \sin n\omega_0 t \, dt$$

The above equation represents trigonometric Fourier series representation of xt

Exponential Fourier Series EFS

Consider a set of complex exponential functions $\left\{e^{jn\omega_0 t}\right\} (n = 0, \pm 1, \pm 2...)$ which is orthogonal over the interval $(t_0, t_0 + T)$. Where $T = \frac{2\pi}{\omega_0}$. This is a complete set so it is possible to represent any function ft as shown below

$$f(t) = F_0 + F_1 e^{j\omega_0 t} + F_2 e^{j2\omega_0 t} + \ldots + F_n e^{jn\omega_0 t} + \ldots$$
$$F_{-1} e^{-j\omega_0 t} + F_{-2} e^{-j2\omega_0 t} + \ldots + F_{-n} e^{-jn\omega_0 t} + \ldots$$

$$\therefore f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \qquad (t_0 < t < t_0 + T) \ldots \ldots (1)$$

Equation 1 represents exponential Fourier series representation of a signal ft over the interval (t_0, t_0+T). The Fourier coefficient is given as

$$F_n = \frac{\int_{t_0}^{t_0+T} f(t) (e^{jn\omega_0 t})^* dt}{\int_{t_0}^{t_0+T} e^{jn\omega_0 t} (e^{jn\omega_0 t})^* dt}$$
$$= \frac{\int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt}{\int_{t_0}^{t_0+T} e^{-jn\omega_0 t} e^{jn\omega_0 t} dt}$$
$$= \frac{\int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt}{\int_{t_0}^{t_0+T} 1\, dt} = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt$$

$$\therefore F_n = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt$$

Relation between Trigonometric and Exponential Fourier Series

Consider a periodic signal x*t*, the TFS & EFS representations are given below respectively

$x(t) = a_0 + \Sigma^{\infty}_{n=1}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t). \ldots \ldots (1)$

$x(t) = \Sigma^{\infty}_{n=-\infty} F_n e^{jn\omega 0t}$

$= F_0 + F_1 e^{j}\omega 0^{t} + F_2 e^{j2}\omega 0^{t} + \ldots + F_n e^{jn}\omega 0^{t} + \ldots$

$_{F-1e}{}^{-j}\omega 0^{t} + {}_{F-2e}{}^{-j2}\omega 0^{t} + \ldots + {}_{F-ne}{}^{-jn}\omega 0^{t} + \ldots$

$= \quad F_0 + F_1 (\cos \omega_0 t + j \sin \omega_0 t) + F_2 (\cos 2\omega_0 t + j \sin 2\omega_0 t) + \ldots + F_n (\cos n\omega_0 t + j \sin n\omega_0 t) + \ldots + F_{-1} (\cos \omega_0 t - j \sin \omega_0 t) + F_{-2} (\cos 2\omega_0 t - j \sin 2\omega_0 t) + \ldots + F_{-n} (\cos n\omega_0 t - j \sin n\omega_0 t) + \ldots$

$= \quad F_0 + (F_1 + F_{-1}) \cos \omega_0 t + (F_2 + F_{-2}) \cos 2\omega_0 t + \ldots + j(F_1 - F_{-1}) \sin \omega_0 t + j(F_2 - F_{-2}) \sin 2\omega_0 t + \ldots$

$\therefore \quad x(t) = F_0 + \sum_{n=1}^{\infty} ((F_n + F_{-n}) \cos n\omega_0 t + j(F_n - F_{-n}) \sin n\omega_0 t) \ldots\ldots (2)$

Compare equation 1 and 2.

$a_0 = F_0$

$a_n = F_n + F_{-n}$

$b_n = j(F_n - F_{-n})$

Similarly,

$F_n = \frac{1}{2}(a_n - jb_n)$

$F_{-n} = \frac{1}{2}(a_n + jb_n)$

3. State and prove the sampling theorem?

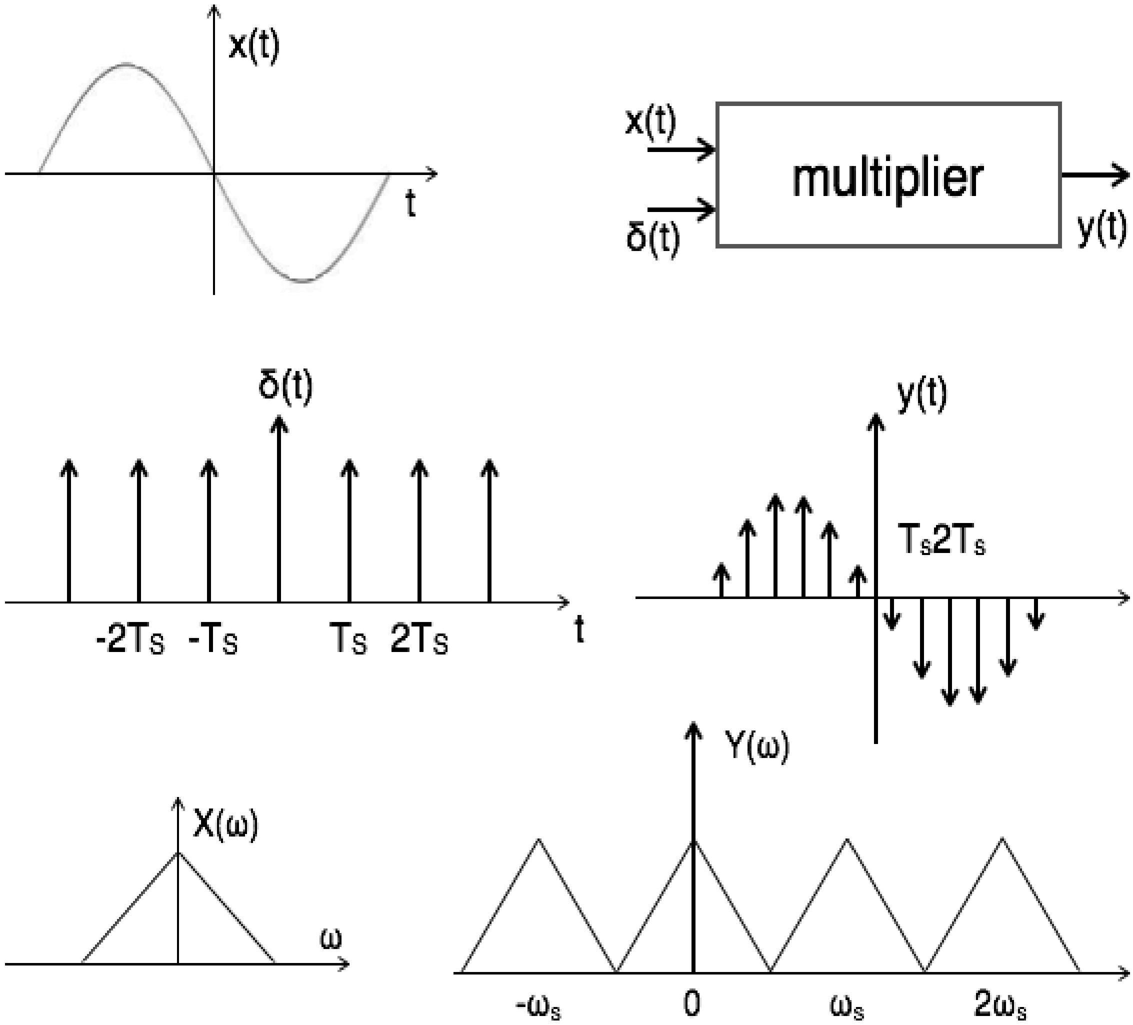
Ans:

Statement: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal. i. e.

$$F_s \geq 2f_m$$

Proof: Consider a continuous time signal x(t). The spectrum of x(t) is a band limited to f_m Hz i.e. the spectrum of x(t) is zero for $|\omega| > \omega_m$.

Sampling of input signal x(t) can be obtained by multiplying x(t) with an impulse train δ(t) of period T_s. The output of multiplier is a discrete signal called sampled signal which is represented with y(t) in the following diagrams:



Here, you can observe that the sampled signal takes the period of impulse. The process of sampling can be explained by the following mathematical expression:

Take Fourier transform on both sides.

$$\text{Sampled signal}\, y(t) = x(t).\,\delta(t) \;\ldots\ldots (1)$$

The trigonometric Fourier series representation of δt is given by

$$\delta(t) = a_0 + \Sigma_{n=1}^{\infty}(a_n \cos n\omega_s t + b_n \sin n\omega_s t) \;\ldots\ldots (2)$$

Where $a_0 = \frac{1}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)dt = \frac{1}{T_s}\delta(0) = \frac{1}{T_s}$

$$a_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\cos n\omega_s\, dt = \frac{2}{T_2}\delta(0)\cos n\omega_s 0 = \frac{2}{T}$$

$$b_n = \frac{2}{T_s}\int_{\frac{-T}{2}}^{\frac{T}{2}} \delta(t)\sin n\omega_s t\,dt = \frac{2}{T_s}\delta(0)\sin n\omega_s 0 = 0$$

Substitute above values in equation 2.

$$\therefore\ \delta(t) = \frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t + 0)$$

Substitute δt in equation 1.

$$\rightarrow y(t) = x(t).\,\delta(t)$$

$$= x(t)[\frac{1}{T_s} + \Sigma_{n=1}^{\infty}(\frac{2}{T_s}\cos n\omega_s t)]$$

$$= \frac{1}{T_s}[x(t) + 2\Sigma_{n=1}^{\infty}(\cos n\omega_s t)x(t)]$$

$$y(t) = \frac{1}{T_s}[x(t) + 2\cos\omega_s t.\,x(t) + 2\cos 2\omega_s t.\,x(t) + 2\cos 3\omega_s t.\,x(t)\ldots\ldots]$$

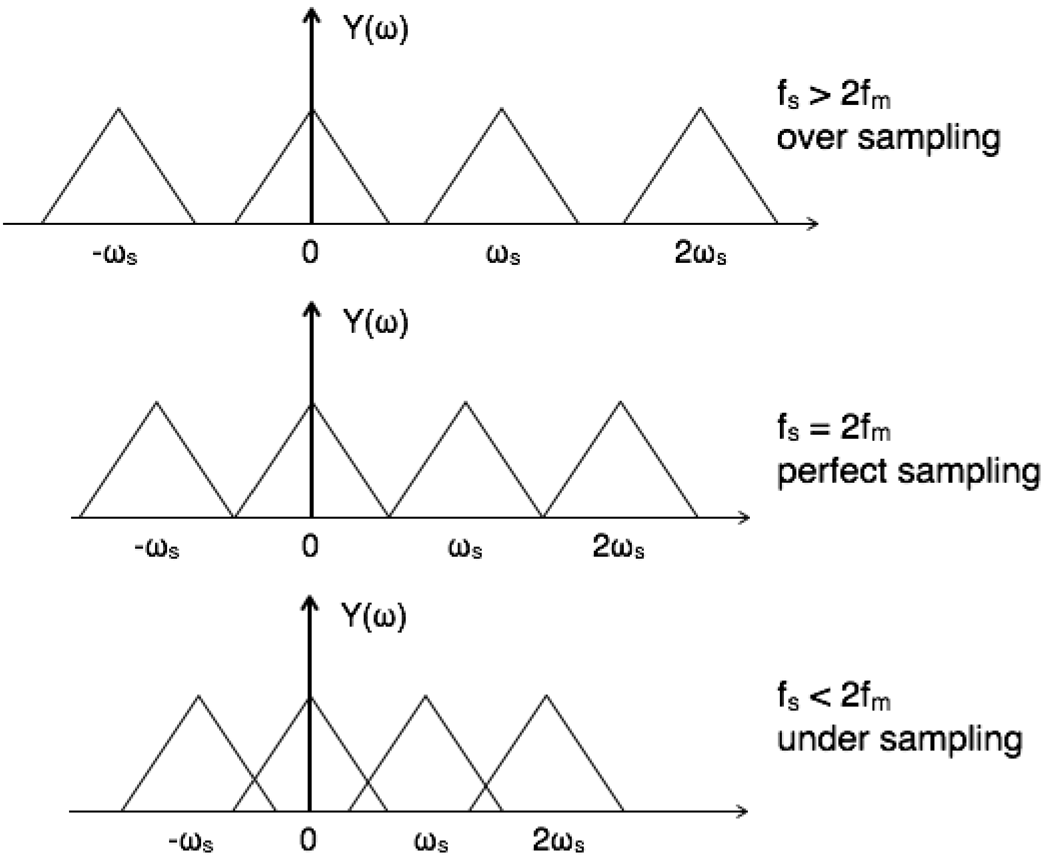
Take Fourier transform on both sides.

$$Y(\omega) = \frac{1}{T_s}[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s)+\ldots]$$

$$\therefore\ Y(\omega) = \frac{1}{T_s}\Sigma_{n=-\infty}^{\infty}X(\omega-n\omega_s) \qquad where\ n = 0,\pm1,\pm2,\ldots$$

To reconstruct x(t), you must recover input signal spectrum X(ω) from sampled signal spectrum Y(ω), which is possible when there is no overlapping between the cycles of Y(ω).

Possibility of sampled frequency spectrum with different conditions is given by the following diagrams:



Aliasing Effect

The overlapped region in case of under sampling represents aliasing effect, which can be removed by

- considering $f_s > 2f_m$
- By using anti aliasing filters.

4. Explain how to derive Fourier transform from Fourier series?

Ans: The main drawback of Fourier series is, it is only applicable to periodic signals. There are some naturally produced signals such as nonperiodic or aperiodic, which we cannot represent using Fourier series. To overcome this shortcoming, Fourier developed a mathematical model to transform signals between time *orspatial* domain to frequency domain & vice versa, which is called 'Fourier transform'.

Fourier transform has many applications in physics and engineering such as analysis of LTI systems, RADAR, astronomy, signal processing etc.

Deriving Fourier transform from Fourier series

Consider a periodic signal f*t* with period T. The complex Fourier series representation of f*t* is given as

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$
$$= \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{T_0}kt} \ldots\ldots(1)$$

Let $\frac{1}{T} = \Delta f$, then equation 1 becomes 0

$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi k\Delta f t} \ldots\ldots(2)$

but you know that

$$a_k = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jk\omega_0 t}\,dt \; 0$$

Substitute in equation 2.

$$2 \Rightarrow f(t) = \Sigma_{k=-\infty}^{\infty} \frac{1}{T_0}\int_{t_0}^{t_0+T} f(t)e^{-jk\omega_0 t}\,dt\, e^{j2\pi k\Delta f t}$$

Let $t_0 = \frac{T}{2}$

$$= \Sigma_{k=-\infty}^{\infty} [\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)e^{-j2\pi k\Delta f t}\,dt]\, e^{j2\pi k\Delta f t}\,.\Delta f$$

In the limit as $T \to \infty$, Δf approaches differential df, $k\Delta f$ becomes a continuous variable f, and summation becomes integration

$$f(t) = lim_{T\to\infty}\left\{\Sigma_{k=-\infty}^{\infty} [\int_{\frac{-T}{2}}^{\frac{T}{2}} f(t)e^{-j2\pi k\Delta f t}\,dt]\, e^{j2\pi k\Delta f t}\,.\Delta f\right\}$$
$$= \int_{-\infty}^{\infty} [\int_{-\infty}^{\infty} f(t)e^{-j2\pi f t}\,dt] e^{j2\pi f t}\,df$$

$$f(t) = \int_{-\infty}^{\infty} F[\omega]e^{j\omega t}d\omega$$

Where $F[\omega] = [\int_{-\infty}^{\infty} f(t)e^{-j2\pi f t}\,dt]$

Fourier transform of a signal

$$f(t) = F[\omega] = [\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt]$$

Inverse Fourier Transform is

$$f(t) = \int_{-\infty}^{\infty} F[\omega]e^{j\omega t}d\omega$$

5. List the properties of Fourier series?

Ans: These are properties of Fourier series:

Linearity Property

If $x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{xn}$ & $y(t) \xleftrightarrow{\text{fourier series coefficient}} f_{yn}$

then linearity property states that

$$a\,x(t) + b\,y(t) \xleftrightarrow{\text{fourier series coefficient}} a\,f_{xn} + b\,f_{yn}$$

Time Shifting Property

If $x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{xn}$

then time shifting property states that

$$x(t - t_0) \xleftrightarrow{\text{fourier series coefficient}} e^{-jn\omega_0 t_0} f_{xn}$$

Frequency Shifting Property

If $x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{xn}$

then frequency shifting property states that

$$e^{jn\omega_0 t_0}\,.\,x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{x(n-n_0)}$$

Time Reversal Property

If $x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{xn}$

then time reversal property states that

If $x(-t) \xleftrightarrow{fourier\ series\ \ coefficient} f_{-xn}$

Time Scaling Property

If $x(t) \xleftrightarrow{fourier\ series\ \ coefficient} f_{xn}$

then time scaling property states that

If $x(at) \xleftrightarrow{fourier\ series\ \ coefficient} f_{xn}$

Time scaling property changes frequency components from ω_0 to $a\omega_0$.

Differentiation and Integration Properties

If $x(t) \xleftrightarrow{fourier\ series\ \ coefficient} f_x$

then differentiation property states that

If $\frac{dx(t)}{dt} \xleftrightarrow{fourier\ series\ \ coefficient} jn\omega_0 . f_{xn}$

& integration property states that

If $\int x(t)dt \xleftrightarrow{\text{fourier series coefficient}} \frac{f_{xn}}{jn\omega_0}$

Multiplication and Convolution Properties

If $x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{xn}$ & $y(t) \xleftrightarrow{\text{fourier series coefficient}} f_{yn}$

then multiplication property states that

$x(t).y(t) \xleftrightarrow{\text{fourier series coefficient}} T f_{xn} * f_{yn}$

& convolution property states that

$x(t) * y(t) \xleftrightarrow{\text{fourier series coefficient}} T f_{xn}.f_{yn}$

Conjugate and Conjugate Symmetry Properties

If $x(t) \xleftrightarrow{\text{fourier series coefficient}} f_{xn}$

Then conjugate property states that

$x*(t) \xleftrightarrow{\text{fourier series coefficient}} f*_{xn}$

Conjugate symmetry property for real valued time signal states that

$$f*_{xn} = f_{-xn}$$

& Conjugate symmetry property for imaginary valued time signal states that

$$f*_{xn} = -f_{-xn}$$

Multiple Choice Questions

1. The inverse Fourier transform of δ(t) is

A.	U(t)	B	1
C	δ(t)	D.	ej2pt

2. The Fourier transform of e^{-2t} for t ≥0 given by

A. 1/(2+jw)

B. 1/(2-jw)

C. 2/(2+jw)

D. 2/(2-jw)

3. Fourier transform of Signum function.
   - A. -j/∏f
   - B. -2j/∏f
   - C. j/∏f
   - D. j/2∏f

4. The trigonometric Fourier series of an even function of time does not have
   - A. dc term
   - B. cosine term
   - C. sine term
   - D. odd harmonic terms

5. The F.T is applicable for
   - A. periodic and non periodic signal
   - B. periodic signal
   - C. non periodic signal
   - D. Periodic or non periodic signal

6. The trigonometric Fourier series of an odd function of time does not have

A. sine term B. cosine term and dc term
C. cosine term D. dc term

7. Minimum sampling rate is called
   - A. Over sampling
   - B. Under sampling
   - C. Sampling time period
   - D. Nyquist rate

8. Any function whose Fourier transform is zero for frequencies outside finite interval is called

B. high pass function
C. low pass function
D. band limited function
E. band pass function

9. Sampled frequency less than nyquist rate is called
A. under sampling
B. over sampling
C. critical sampling
D. nyquist sampling

10 .Effect caused by under sampling is called
A. smoothing
B. sharpening
C. summation
D. aliasing

Key

1.	b
2.	a
3.	a
4.	c
5.	a
6.	b
7.	d
8.	c
9.	a
10.	d

FILL IN THE BLANKS

1. Product of two functions in time domain is what, in frequency domain_______

2. Band limited function can be recovered from its samples if acquired samples are at rate twice highest frequency, this theorem is called_____________
3. _____ filter is known anti aliasing filter in the sampling
4. For ________signals Fourier serious is applicable
5. The plot of magnitude versus frequency is called ________
6. The plot of phase versus frequency is called ________
7. To avoid aliasing reduce ___
8. If the signal is compressed in the time domain, it will be________ in the frequency domain
9. Symmetry which is hidden in the periodic signal is called ___________symmetry
10. The exponential Fourier series coefficient C_n=___________

Key

1	Convolution
2	sampling theorem
3	Low pass filter
4	Periodic signals
5	Magnitude spectrum
6	Phase spectrum
7	Reduce the bandwidth
8	Expanded
9	Hidden
10	$(a_n-jb_n)/2$

UNIT-III

2 MARKS QUESTIONS WITH ANSWERS

1. Define z-transform and inverse z-transform.

Ans: The z-transform of a general discrete-time signal x(n) is defined as,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

It is used to represent complex sinusoidal discrete time signal.

The inverse z-transform is given by,

$$x(n) = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

2. Define the Frequency Shifting Property of Laplace transform

Ans:

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$
Then frequency shifting property states that

$$e^{s_0 t}.x(t) \overset{L.T}{\longleftrightarrow} X(s - s_0)$$

3. State convolution property.

Ans: Convolution property states that if $x_1(n) \overset{z}{\longleftrightarrow} X_1(z)$ and

$x_2(n) \overset{z}{\longleftrightarrow} X_2(z)$

Then $x_1(n) * x_2(n) \overset{z}{\longleftrightarrow} X_1(z)X_2(z)$

4. What are the methods used to find inverse z-transform?

Ans:

- Long division method (or) Power series expansion
- Residue method
- Partial fraction method
- Convolution method

5. What do you mean by ROC? (or) Define Region of convergence.

Ans:

By definition, $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$ --------------- (1)

The z-transform exists when the infinite sum in equation (1) converges. A necessary condition for convergence is absolute summability of $x(n)z^{-n}$. The value of z for which the z-transform converges is called region of convergence.

3 MARKS QUESTIONS WITH ANSWERS

1. List the properties of region of convergence for the z-transform.

Ans:

- The ROC of X(z) consists of a ring in the z-plane centered about the origin.
- The ROC does not contain any poles.
- If x(n) is of finite duration then the ROC is the entire z-plane except z = 0 and / or z = ∞.
- If x(n) is a right sided sequence and if the circle $|z| = r_0$ is in the ROC then all finite values of z for which $|z| > r_0$ will also be in the ROC.
- If x(n) is a left sided sequence and if the circle $|z| = r_0$ is in the ROC then all values of z for which $0 < |z| < r_0$ will also be in the ROC.

· If x(n) is two sided sequence and if the circle $|z| = r_0$ is in the ROC then the ROC will consist of a ring in the z-plane that includes the circle $|z| = r_0$.

2. List the properties of z-transform.

Ans:

- Linearity
- Time shifting
- Time reversal
- Time scaling
- Multiplication
- Convolution
- Parseval's relation

3. Determine the z-transform of unit step sequence.

Ans:

The unit step sequence is given

$$u(n) = \begin{cases} 1; & n \geq 0 \\ 0; & \text{otherwise} \end{cases}$$

Here x(n) = u(n)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 1^n u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (z^{-1})^n$$

$$X(z) = \frac{1}{1-z^{-1}}; |z| > 1$$

The ROC is at z > 1 ie., the area outside the unit circle

4. How the stability of a system can be found in z-transform?

Ans: Let h(n) be a impulse response of a causal or non-causal LTI system and H(z) be the z-transform ofh(n). The stability of a system can be found from ROC using the theorem which states,

'A Linear time system with the system function H(z) is BIBO stable if and only if the ROC for H(z) contains the unit circle.

The condition for the stable system is, $\sum_{n=-\infty}^{\infty} h(n) < \infty$

5. What is the condition for causality in terms of z-transform?

Ans: The condition for Linear time invariant system to be causal is given as,

h(n) = 0, n<0. Where h(n) is the sample response of the system.

When the sequence is causal , its ROC is the exterior of the circle. i.e., |z|>r. LTI system is causal if and only if the ROC of the system function is exterior of the circle.

5 MARKS QUESTIONS WITH ANSWERS

1. State and prove initial value theorem of z-transform.

Ans: If x(n) is a causal sequence then its initial value is given by

$$x(0) = \lim_{z \to \infty} X(z)$$

Proof:

By definition of z-transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

x(n) = 0 for n<0, since x(n) is a causal sequence.

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$= x(0)+x(1)z^{-1}+x(2)z^{-2}+\ldots\ldots$$

Applying limit z→∞ on both sides,

$$\lim_{z \to \infty} X(z) = x(0)$$

2. What is the transfer function of a system whose poles are at -0.3±j0.4 and a zero at -0.2?

Ans: Poles are at z = -0.3±j0.4

∴ Denominator polynomial = [z-(-0.3+j0.4)][z-(-0.3-j0.4)] Zero at z = -0.2

∴ Numerator polynomial = z+0.2 The transfer function is given by, Poles are at z = -0.3±j0.4

∴ Denominator polynomial = [z-(-0.3+j0.4)][z-(-0.3-j0.4)]

Zero at z = -0.2

∴ Numerator polynomial = z+0.2

The transfer function is given by,

$$H(z) = \frac{z+0.2}{(z+0.3-j0.4)(z+0.3+j0.4)}$$

$$= \frac{z+0.2}{(z+0.3)^2+(0.4)^2}$$

$$H(z) = \frac{z+0.2}{z^2+0.6z+0.25}$$

3. Determine the Laplace transform and ROC for the basic signals?

Ans:

ft	**F**s	**ROC**
$u(t)$	$\frac{1}{s}$	ROC: Re{s} > 0
$t\,u(t)$	$\frac{1}{s^2}$	ROC:Re{s} > 0
$t^n\,u(t)$	$\frac{n!}{s^{n+1}}$	ROC:Re{s} > 0
$e^{at}\,u(t)$	$\frac{1}{s-a}$	ROC:Re{s} > a

$e^{-at}\,u(t)$	$\frac{1}{s+a}$	ROC:Re{s} > -a
$e^{at}\,u(t)$	$-\frac{1}{s-a}$	ROC:Re{s} < a
$e^{-at}\,u(-t)$	$-\frac{1}{s+a}$	ROC:Re{s} < -a
$t\,e^{at}\,u(t)$	$\frac{1}{(s-a)^2}$	ROC:Re{s} > a
$t^n\,e^{at}\,u(t)$	$\frac{n!}{(s-a)^{n+1}}$	ROC:Re{s} > a
$t\,e^{-at}\,u(t)$	$\frac{1}{(s+a)^2}$	ROC:Re{s} > -a

4. list the properties of Laplace transform?

Ans: The properties of Laplace transform are:

6) **Linearity Property**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

& $y(t) \overset{L.T}{\longleftrightarrow} Y(s)$

Then linearity property states that

$$ax(t) + by(t) \overset{L.T}{\longleftrightarrow} aX(s) + bY(s)$$

7) **Time Shifting Property**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then time shifting property states that

$$x(t - t_0) \overset{L.T}{\longleftrightarrow} e^{-st_0}\,X(s)$$

8) **Frequency Shifting Property**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then frequency shifting property states that

$$e^{s_0 t}.\,x(t) \overset{L.T}{\longleftrightarrow} X(s-s_0)$$

9) **Time Reversal Property**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then time reversal property states that

$$x(-t) \overset{L.T}{\longleftrightarrow} X(-s)$$

10) **Time Scaling Property**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then time scaling property states that

$$x(at) \overset{L.T}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{s}{a}\right)$$

11) **Differentiation and Integration Properties**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

Then differentiation property states that

$$\frac{dx(t)}{dt} \overset{L.T}{\longleftrightarrow} s.\,X(s)$$

$$\frac{d^n x(t)}{dt^n} \overset{L.T}{\longleftrightarrow} (s)^n.\,X(s)$$

The integration property states that

$$\int x(t)dt \overset{L.T}{\longleftrightarrow} \frac{1}{s} X(s)$$

$$\iiint \ldots \int x(t)dt \overset{L.T}{\longleftrightarrow} \frac{1}{s^n} X(s)$$

12) **Multiplication and Convolution Properties**

If $x(t) \overset{L.T}{\longleftrightarrow} X(s)$

and $y(t) \overset{L.T}{\longleftrightarrow} Y(s)$

Then multiplication property states that

$$x(t).\,y(t) \overset{L.T}{\longleftrightarrow} \frac{1}{2\pi j} X(s) * Y(s)$$

The convolution property states that

$$x(t) * y(t) \overset{L.T}{\longleftrightarrow} X(s).\,Y(s)$$

5. Determine the Z-Transform of Basic Signals?

Ans:

Z-Transform of Basic Signals

xt	X[Z]
δ	1
$u(n)$	$\frac{Z}{Z-1}$
$u(-n-1)$	$-\frac{Z}{Z-1}$
$\delta(n-m)$	z^{-m}
$a^n u[n]$	$\frac{Z}{Z-a}$
$a^n u[-n-1]$	$-\frac{Z}{Z-a}$
$n\,a^n u[n]$	$\frac{aZ}{\lvert Z-a\rvert^2}$
$n\,a^n u[-n-1]$	$-\frac{aZ}{\lvert Z-a\rvert^2}$
$a^n \cos \omega n u[n]$	$\frac{Z^2-aZ\cos\omega}{Z^2-2aZ\cos\omega+a^2}$
$a^n \sin \omega n u[n]$	$\frac{aZ\sin\omega}{Z^2-2aZ\cos\omega+a^2}$

MULTIPLE CHOICE QUESTIONS

1.The final value theorem is used to find the

A. Steady state value of the system out put

B. initial value of the system out put

C. Transient behavior of the system out put

D. None of these

2. Region of Convergence(ROS) of causal sequence

A. Entire z plane except z=0

B. Entire z plane except z=infinite

C. Entire z plane except z=0 and z= infinite

D. none

3. The ROC of sequence $x[n] = (0.8)^n \cup[n] + (0.4)^n \cup[n]$

A. $|z| > 0.8$

B. $|z| > 0.4$

C. $0.4 < |z| < 0.8$

D. $|z| < 0.8$

4. The Laplace transform of impulse $\delta(t)$ is

A. 1

B. 1/s

C. S

D. $1/s^2$

5.If Laplace transform of f(t) is F(s), then £ f(t - a) u (t - a)= 0

A. e^{as} F(s)

B. e^{-as} F(s)

C. - e^{as} F(s)

D. - e^{-as} F(s)

6. Z transformer of $[\ a(x_k) + b(y_k)\ ]=$

A. aX(z) - bY(z)

B. aX(z) + bY(z)

C. aX(z) + bY(z) + a/b

D. aX(z) + bY(z) + bY(z) - a/b

7. Which of the following method is used to determine the inverse z transform

A. Power series method

B. Partial fraction method

C. Residue method

D. All the above

8. If $L[f(t)] = \frac{2(s+1)}{s^2+2s+5}$ then, $f(0^+)$ and $f(\infty)$ are given by

A. 0 and 2

B. 2, 0

C. 0, 1

D. $\frac{2}{5}$, 0

9.The Laplace transform of the e^{at} u(t) is

A. 1/(s+a)

B. 1/(s-a)

C. 2/(s+a)

D. 2/(s+a)

10 Laplace transform of e^{at} cos t) is

A. $\frac{s-a}{(s-a)^2+\alpha^2}$

B. $\frac{s+a}{(s+a)^2+\alpha^2}$

C. $\frac{1}{(s-a)^2}$

D. none of these

Key

1. a
2. a
3. a
4. a
5. b
6. b

7. d

8. b

9. b

10. a

FILL IN THE BLANKS

1.The inverse Laplace transform of $\frac{1}{(s+1)^3}$ is_____________

2. If the system transfer function of a discrete time system $H(z) = \frac{z}{z-1}$ then system is_____________

3. A system is stable if ROC ____

4. The number of possible regions of convergence of the function $\frac{(e^{-2}-2)z}{(z-e^{-2})(z-2)}$ is____

5. The impulse response of a system described by the differential equation $\frac{d^2y}{dt^2} + y(t) = x(t)$ will be_______

6. The function $\frac{\sin(\pi u)}{(\pi u)}$ is denoted by_____

7. z-transform converts convolution of time-signals to _________

8. Zero-order hold used in practical reconstruction of continuous-time signals is mathematically represented as a weighted-sum of rectangular pulses shifted by:______

9. The region of convergence of the z-transform of the signal x(n) ={2, 1, 1, 2} n = 0 is _______

10. When two honest coins are simultaneously tossed, the probability of two heads on any given trial is:________

Key:

1	$\frac{t^2}{2}e^{-t}$

2	Stable
3	include the unit circle
4	1
5	a sinusoidal
6	Sinc
7	Multiplication
8	Integer multiples of the sampling interval.
9	all z, except z = 0 and z = ∞
10	¼

UNIT-IV

2 MARKS QUESTIONS WITH ANSWERS

1. Define Probability?
Ans: Probability of an event is defined as
Probability of an event happening= No of ways it can happen/total no of outcomes.

2. Define a random variable?
Ans: random variable is a variable whose value is unknown or a function that assigns value to each of an experiment's outcome.

3. Define a Sample Space?
Ans: The sample space of an experiment is the set of all possible outcomes of that experiment.

4. Define bayes theorem?
Ans: The bayes theorem or bayes rule describes the probability if an event, based on prior knowledge of the conditions that might be related to the event.

5. Define probability density function?
Ans: The probability density function is the derivative of the probability distribution, and it id denoted by FX(x).
Fx(x) =d/dx Fx(x).

3 MARKS QUESTIONS WITH ANSWERS

1. Define statistical independence.
Ans: The two events A and B are statistically independent if and only if
P (AnB) = P (A).P (B). Similarly X and Y are statistically independent random variables if and only if P $\{X\leq x, Y\leq y\} = P\{X\leq x\}\ P\{Y\leq y\}$
Fxy(y)=Fx(x).Fy(y)
Fx(y)=fx(x)fy(y)

2. Defline Radom Procès.
Ans: A random variable, x (ζ), can be defined from a Random event, ζ, by assigning values xi to each possible outcome, Ai, of the event. Next define a Random Process, x(ζ) ,t , a function

of both the event and time, by assigning to each outcome of a random event, ζ , a function in time, xi(t) , chosen from a set of functions, xi(t) .

3. Define Central Limit Theorem.

Ans: The central limit theorem states that the random variable X which is the sum of the large number of random variables always approaches the Gaussian distribution irrespective of the type of distribution each variable process and their amount of contribution into the sum.
X3=X1+X2

4. Explain deterministic and non-deterministic process.

Ans:

1. If the future values of any sample function can't be predicted exactly from the observed past values it is called non-deterministic.
2. If the future values of any sample function can be predicted exactly from the observed past values it is called deterministic.

5. Define compound probability theorem

Ans: If the probability of event A happening as a result of a trail is P (A) and after A has happened**,** the probability of a event B happening as a result of another trail is P (B/A), then the probability of both the events happening as a result of two trails is P (AB) or P (AnB) =P (A).P (B/A).

5 MARKS QUESTIONS WITH ANSWERS

1.Define random process? And discuss the classifications of random processes?

Ans: Introduction

A Random Variable 'X' is defined as a function of the possible outcomes'**s '** of an experiment or whose value is unknown and possibly depends on a set of random events. It is denoted by X(s).

The Concept of Random Process is based on enlarging the random variable concept to include time't' and is denoted by X(t,s) i.e., we assign a time function to every outcome according to some rule.In short, it is represented as X(t). A random process clearly represents a family or ensemble of time functions when t and s are variables. Each member time function is called a sample function or ensemble member.

Depending ontime 't' and outcome' s' fixed or variable, A random process represents a single time function when t is a variable and s is fixed at a specific value.

A random process represents a random variable when t is fixed and s is a variable A random process represents a number when t and s are both fixed

Classification of Random Processes

A Random ProcessesX(t) has been classified in to four types as listed below depending on whether random variable X and time t is continuous or discrete.

1.Continuous Random Processes

If a random variable X is continuous and time t can have any of a continuum of values, then X(t) is called as a continuous random process.

Example: Thermal Noise

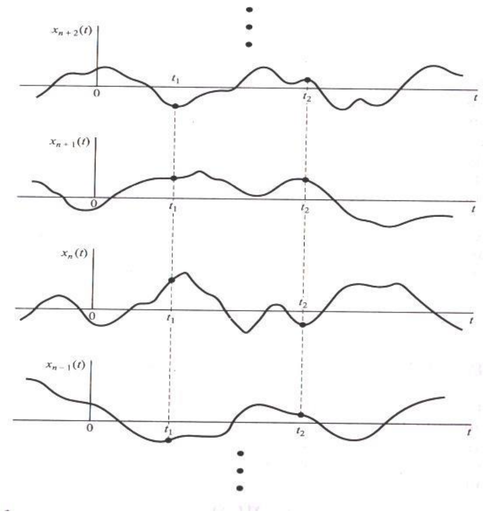


Fig 1: Continuous Random Processes

2.Discrete Random Processes

If a random variable X is discrete and time t is continuous, then X(t) is called as a discrete random process. The sample functions will have only two discrete values

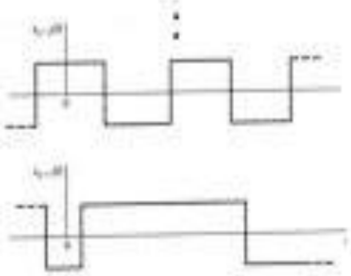


Fig 2: Discrete Random Processes

3.Continuous random Sequence

If a random variable X is continuous and time t is discrete, then X(t) is called as a continuous random sequence. Since a continuous random sequence is defined at only discrete times, it is also called as discrete time random process. It can be generated by periodically sampling the ensemble members of continuous random processes. These types of processes are important in the analysis of digital signal processing systems.

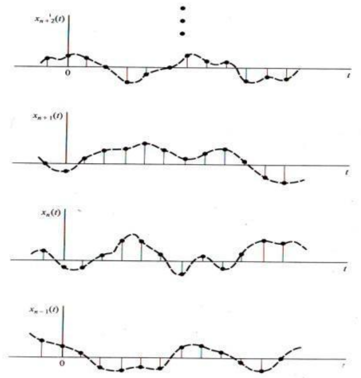


Fig 3: Continuous Random Sequence

4.Discrete Random Sequence

If a random variable X and time t areboth discrete, then X(t) is called as a discrete random sequence. It can be generated by sampling the sample functions of discrete random process or rounding off the samples of continuous random sequence.

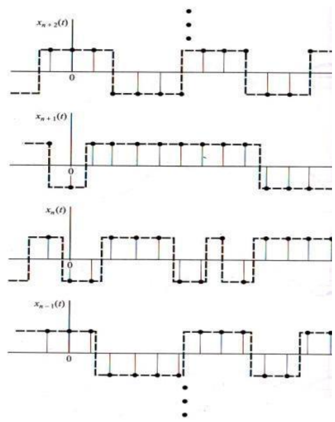


Fig 4: Discrete Random Sequence Deterministic and Non-deterministic processes

In addition to the processes, discussed above a random process can be described by the form of its sample functions.

A Process is said to be deterministic process, if future values of any sample function can be predicted from past values. These are also called as regular signals,which have a particular shape.

Example: $X(t) = A \sin(\omega t + \Theta)$, A, ω and Θ may be random variables

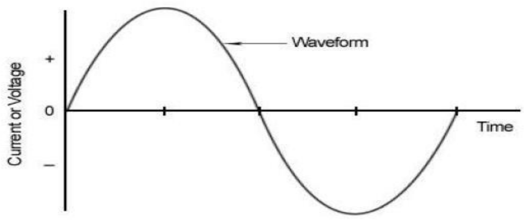


Fig 5: Example of Deterministic Process

A Process is said to be non-deterministic process, if future values of any sample function cannot be predicted from past values.

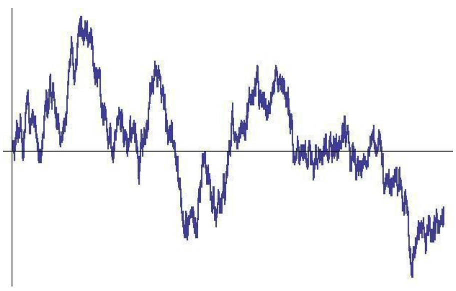


Fig 6: Example of Non-Deterministic Processes

2. Define the Distribution Function and Density function of random process?

Ans:

Distribution Function:

Probability distribution function (PDF) which is also be called as Cumulative Distribution Function (CDF) of a real valued random variable 'X ' is the probability that X will take value less than or equal to X.

It is given by

$$F_X\ (x) = P\{X \le x\}$$

In case of random process X(t), for a particular time t, the distribution function associated with the random variable X is denoted as

$$F_X\ (x: t) = P\{X(t) \le x\}$$

In case of two random variables, X1 = X(t1) and X2 = X (t2), the second order joint distribution function is two dimensional and given by

$$F_X\ \left(x_{1,}, x_2 : t_{1,}, t_2\right) = P\{X(t_1) \le x_{1,}, X(t_2) \le x_2\ \}$$

and can be similarly extended to N random variables, called as Nth order joint distribution function

Density Function:

The probability density function(pdf) in case of random variable is defined as the derivative of the distribution function and is given by

$$f_X\ (x) = \frac{dF_x\ (x)}{dx}$$

In case of random process, density function is given by

$$f_X\ (x: t) = \frac{dF_x\ (x: t)}{dx}$$

In case of two random functions, two dimensional density function is given by

$$f_X\ \left(x_{1,}, x_2 : t_{1,}, t_2\right) = \frac{\partial^2 F_X\ \left(x_{1,}, x_2 : t_{1,}, t_2\right)}{\partial x_1 \partial x_2}$$

3. **Discuss the Statistical properties of Random Processes?**

Ans:

Statistical properties of Random Processes: The following are the statistical properties of random processes.

% **Mean:** The mean value of a random process X(t) is equal to the expected value of the random process i.e. $\bar{X}(t) = E[X(t)] = \int$

% **Autocorrelation:** Consider random process X(t). Let X_1 and X_2 be two random variables defined at times t_1 and t_2 respectively with joint density function

$f_X(x_1, x_2 ; t_1, t_2)$. The correlation of X_1 and X_2, $E[X_1 X_2] = E[X(t_1) X(t_2)]$ is called the autocorrelation function of the random process X(t) defined as

$R_{XX}(t_1,t_2) = E[X_1 X_2] = E[X(t_1) X(t_2)]$ or

$$R_{XX}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_X(x1, x2\ ;\ t1, t2)\, dx_1 dx_2$$

% **Cross correlation:** Consider two random processes X(t) and Y(t) defined with random variables X and Y at time instants t_1 and t_2 respectively. The joint density function is $f_{xy}(x,y ; t_1,t_2)$.Then the correlation of X and Y, $E[XY] = E[X(t_1) Y(t_2)]$ is called the cross correlation function of the random processes X(t) and Y(t) which is

defined as

$R_{XY}(t_1,t_2) = E[X Y] = E[X(t_1) Y(t_2)]$ or $R_{XY}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f_{x,y}(x, y\ ;\ t1, t2)\, dxdy$

4. Explain the different types of Stationary Processes?

Ans: **Stationary Processes:** A random process is said to be stationary if all its statistical properties such as mean, moments, variances etc… do not change with time. The stationarity which depends on the density functions has different levels or orders.

1. **First order stationary process:** A random process is said to be stationary to order one or first order stationary if its first order density function does not change with time or shift in time value. If X(t) is a first order stationary process then
   $f_X(x_1;t_1) = f_X(x_1;t_1+\Delta t)$ for any time t_1. Where Δt is shift in time value. Therefore the

   condition for a process to be a first order stationary random process is that its mean value must be constant at any time instant. i.e. $E[X(t)] = \bar{X}$ = constant.

2. **Second order stationary process:** A random process is said to be stationary to order

two or second order stationary if its second order joint density function does not change with time or shift in time value i.e. $f_X(x_1, x_2 ; t_1, t_2) = f_X(x_1, x_2;t_1+\Delta t, t_2+\Delta t)$ for all t_1,t_2 and Δt. It is a function of time difference (t_2, t_1) and not absolute time t. Note that a second order stationary process is also a first order stationary process. The condition for a process to be a second order stationary is that its autocorrelation

should depend only on time differences and not on absolute time. i.e. If

$R_{XX}(t_1,t_2) = E[X(t_1) X(t_2)]$ is autocorrelation function and $\tau = t_2 - t_1$ then

$R_{XX}(t_1,t_1+ \tau) = E[X(t_1) X(t_1+ \tau)] = R_{XX}(\tau)$. $R_{XX}(\tau)$ should be independent of time t.

3. **Wide sense stationary (WSS) process:** If a random process X(t) is a second order stationary process, then it is called a wide sense stationary (WSS) or a weak sense

stationary process. However the converse is not true. The condition for a wide sense stationary process are

1. $E[X(t)] = \bar{} =$ constant.

2. $E[X(t) X(t+\tau)] = R_{XX}(\tau)$ is independent of absolute time t.

Joint wide sense stationary process: Consider two random processes X(t) and Y(t). If

they are jointly WSS, then the cross correlation function of X(t) and Y(t) is a function of time difference $\tau = t_2 - t_1$only and not absolute time.

i.e. $R_{XY}(t_1,t_2) = E[X(t_1) Y(t_2)]$ .If $\tau = t_2 - t_1$ then $R_{XY}(t,t+ \tau) = E[X(t) Y(t+ \tau)] = R_{XY}(\tau)$. Therefore the conditions for a process to be joint wide sense stationary are

1. $E[X(t)] = \bar{}$constant.

2. $E[Y(t)] = \bar{} =$ constant 3. $E[X(t) Y(t+ \tau)] = R_{XY}(\tau)$ is independent of time t.

4. **Strict sense stationary (SSS) processes:** A random process X(t) is said to be strict Sense stationary if its Nth order joint density function does not change with time or shift in time value. i.e.

$f_X(x_1, x_2 \ldots\ldots x_N ; t_1, t_2, \ldots\ldots t_N) = f_X(x_1, x_2 \ldots\ldots x_N ; t_1+\Delta t, t_2+\Delta t, \ldots t_N+\Delta t)$ for all $t_1, t_2 \ldots t_N$ and Δt. A process that is stationary to all orders n=1,2,. . . N is called strict sense stationary process. Note that SSS process is also a WSS process. But the reverse is not true.

5.Define Ergodic process? And the properties of autocorrelation and cross correlation functions?

Ans:

Ergodic Theorem and Ergodic Process: The Ergodic theorem states that for any random process X(t), all time averages of sample functions of x(t) are equal to the corresponding statistical or ensemble averages of X(t). i.e. $\bar{x} = \bar{X}$ or $\mathcal{R}_{xx}(\tau) = R_{XX}(\tau)$. Random processes that satisfy the Ergodic theorem are called Ergodic processes.

Joint Ergodic Process: Let X(t) and Y(t) be two random processes with sample functions x(t) and y(t) respectively. The two random processes are said to be jointly Ergodic if they are individually Ergodic and their time cross correlation functions are equal to their respective statistical cross correlation functions. i.e. 1. $\bar{x} = \bar{X}$, $\bar{y} = \bar{Y}$ 2. $\mathcal{R}_{xx}(\tau) = R_{XX}(\tau)$, $\mathcal{R}_{xy}(\tau) = R_{XY}(\tau)$ and $\mathcal{R}_{yy}(\tau) = R_{YY}(\tau)$.

Mean Ergodic Random Process: A random process X(t) is said to be mean Ergodic if time average of any sample function x(t) is equal to its statistical average, $\bar{X}$ which is constant and the probability of all other sample functions is equal to one. i.e. $E[X(t)] = \bar{X} = A[x(t)] = \bar{x}$ with probability one for all x(t).

Autocorrelation Ergodic Process: A stationary random process X(t) is said to be Autocorrelation Ergodic if and only if the time autocorrelation function of any sample function x(t) is equal to the statistical autocorrelation function of X(t). i.e. $A[x(t)\ x(t+\tau)] = E[X(t)\ X(t+\tau)]$ or $\mathcal{R}_{xx}(\tau) = R_{XX}(\tau)$.

Cross Correlation Ergodic Process: Two stationary random processes X(t) and Y(t) are said to be cross correlation Ergodic if and only if its time cross correlation function of sample functions x(t) and y(t) is equal to the statistical cross correlation function of X(t) and Y(t). i.e. $A[x(t)\ y(t+\tau)] = E[X(t)\ Y(t+\tau)]$ or $\mathcal{R}_{xy}(\tau) = R_{XY}(\tau)$.

Properties of Autocorrelation function: Consider that a random process X(t) is at least WSS and is a function of time difference $\tau = t_2 - t_1$. Then the following are the properties of the autocorrelation function of X(t).

1. Mean square value of X(t) is $E[X^2(t)] = R_{XX}(0)$. It is equal to the power (average) of the process, X(t).
   Proof: We know that for X(t), $R_{XX}(\tau) = E[X(t)\, X(t+\tau)]$. If $\tau = 0$, then $R_{XX}(0) = E[X(t)\, X(t)] = E[X^2(t)]$ hence proved.
2. Autocorrelation function is maximum at the origin i.e. $|R_{XX}(\tau)| \leq R_{XX}(0)$.
   Proof: Consider two random variables $X(t_1)$ and $X(t_2)$ of X(t) defined at time intervals t_1 and t_2 respectively. Consider a positive quantity $[X(t_1) \pm X(t_2)]^2 \geq 0$
   Taking Expectation on both sides, we get $E[X(t_1) \pm X(t_2)]^2 \geq 0$
   $E[X^2(t_1) + X^2(t_2) \pm 2X(t_1)\, X(t_2)] \geq 0$
   $E[X^2(t_1)] + E[X^2(t_2) \pm 2E[X(t_1)\, X(t_2)] \geq 0$
   $R_{XX}(0) + R_{XX}(0) \pm 2\, R_{XX}(t_1, t_2) \geq 0$ [Since $E[X^2(t)] = R_{XX}(0)$]
   Given X(t) is WSS and $\tau = t_2 - t_1$.
   Therefore $2\, R_{XX}(0 \pm 2\, R_{XX}(\tau) \geq 0$
   $R_{XX}(0 \pm R_{XX}(\tau) \geq 0$ or
   $|R_{XX}(\tau)| \leq R_{XX}(0)$ hence proved.
3. $R_{XX}(\tau)$ is an even function of τ i.e. $R_{XX}(-\tau) = R_{XX}(\tau)$.
   Proof: We know that $R_{XX}(\tau) = E[X(t)\, X(t+\tau)]$
   Let $\tau = -\tau$ then
   $R_{XX}(-\tau) = E[X(t)\, X(t-\tau)]$
   Let $u = t - \tau$ or $t = u + \tau$
   Therefore $R_{XX}(-\tau) = E[X(u+\tau)\, X(u)] = E[X(u)\, X(u+\tau)]$

   $R_{XX}(-\tau) = R_{XX}(\tau)$ hence proved.
4. If a random process X(t) has a non zero mean value, $E[X(t)] \neq 0$ and Ergodic with no periodic components, then $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \bar{X}^2$.
   Proof : Consider a random variable X(t)with random variables $X(t_1)$ and $X(t_2)$. Given the mean value is $E[X(t)] = \bar{X} \neq 0$. We know that
   $R_{XX}(\tau) = E[X(t)X(t+\tau)] = E[X(t_1)\ X(t_2)]$. Since the process has no periodic components, as $|\tau| \to \infty$, the random variable becomes independent, i.e.
   $\lim_{|\tau|\to\infty} R_{XX}(\tau) = E[X(t_1)\, X(t_2)] = E[X(t_1)]\, E[X(t_2)]$
   Since X(t) is Ergodic $E[X(t_1)] = E[X(t_2)] = \bar{X}$
   Therefore $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \bar{X}^2$ hence proved.

5. If X(t) is periodic then its autocorrelation function is also periodic.
   Proof: Consider a Random process X(t) which is periodic with period T_0
   Then $X(t) = X(t\pm T_0)$ or
   $X(t+\tau) = X(t+\tau \pm T_0)$. Now we have $R_{XX}(\tau) = E[X(t)X(t+\tau)]$ then
   $R_{XX}(\tau\pm T_0) = E[X(t)X(t+\tau\pm T_0)]$
   Given X(t) is WSS, $R_{XX}(\tau\pm T_0) = E[X(t)X(t+\tau)]$
   $R_{XX}(\tau\pm T_0) = R_{XX}(\tau)$
   Therefore $R_{XX}(\tau)$ is periodic hence proved.
6. If X(t) is Ergodic has zero mean, and no periodic components then
   $\lim_{|\tau|\to\infty} R_{XX}(\tau) = 0$.
   Proof: It is already proved that $\lim_{|\tau|\to\infty} R_{XX}(\tau) = \bar{X}^2$. Where $\bar{X}$ is the mean value of X(t) which is given as zero.
   Therefore $\lim_{|\tau|\to\infty} R_{XX}(\tau) = 0$ hence proved.
7. The autocorrelation function of random process $R_{XX}(\tau)$ cannot have any arbitrary shape.
   Proof: The autocorrelation function $R_{XX}(\tau)$ is an even function of τ and has maximum value at the origin. Hence the autocorrelation function cannot have an arbitrary shape hence proved.
8. If a random process X(t) with zero mean has the DC component A as Y(t) =A + X(t), Then $R_{YY}(\tau) = A^2+ R_{XX}(\tau)$.
   Proof: Given a random process Y(t) =A + X(t).
   We know that $R_{YY}(\tau) = E[Y(t)Y(t+\tau)] =E[(A + X(t)) (A + X(t+ \tau))]$
   $= E[(A^2 + AX(t) + AX(t+ \tau)+ X(t) X(t+ \tau)]$
   $= E[(A^2] + AE[X(t)] + E[AX(t+ \tau)]+ E[X(t) X(t+ \tau)]$
   $=A^2+0+0+ R_{XX}(\tau)$.
   Therefore $R_{YY}(\tau) = A^2+ R_{XX}(\tau)$ hence proved.
9. If a random process Z(t) is sum of two random processes X(t) and Y(t) i.e, Z(t) = X(t) + Y(t). Then $R_{ZZ}(\tau) = R_{XX}(\tau)+ R_{XY}(\tau)+ R_{YX}(\tau)+ R_{YY}(\tau)$
   Proof: Given Z(t) = X(t) + Y(t).
   We know that $R_{ZZ}(\tau) = E[Z(t)Z(t+\tau)]$
   $= E[(X(t)+Y(t)) (X(t+\tau)Y(t+\tau))]$
   $= E[(X(t) X(t+\tau)+ X(t) Y(t+\tau) +Y(t) X(t+\tau) +Y(t) Y(t+\tau))]$
   $= E[(X(t) X(t+\tau)]+ E[X(t) Y(t+\tau)] +E[Y(t) X(t+\tau)] +E[Y(t) Y(t+\tau))]$
   Therefore $R_{ZZ}(\tau) = R_{XX}(\tau)+ R_{XY}(\tau)+ R_{YX}(\tau)+ R_{YY}(\tau)$ hence proved.

Properties of Cross Correlation Function: Consider two random processes X(t) and Y(t) are at least jointly WSS. And the cross correlation function is a function of the time difference $\tau = t_2-t_1$. Then the following are the properties of cross correlation function.

1. $R_{XY}(\tau) = R_{YX}(-\tau)$ is a Symmetrical property.
   Proof: We know that $R_{XY}(\tau) = E[X(t) Y(t+ \tau)]$ also
   $R_{YX}(\tau) = E[Y(t) X(t+ \tau)]$
   Let $\tau = - \tau$ then

$R_{YX}(-\tau) = E[Y(t)\ X(t-\tau)]$
Let $u=t-\tau$ or $t=u+\tau$. then
$R_{YX}(-\tau) = E[Y(u+\tau)\ X(u)] = E[X(u)\ Y(u+\tau)]$
Therefore $R_{YX}(-\tau) = R_{XY}(\tau)$ hence proved.

2. If $R_{XX}(\tau)$ and $R_{YY}(\tau)$ are the autocorrelation functions of X(t) and Y(t) respectively then the cross correlation satisfies the inequality: $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$.
Proof: Consider two random processes X(t) and Y(t) with auto correlation functions $R_{XX}(\tau)$ and $R_{YY}(\tau)$. Also consider the inequality
$E[\frac{X(t)}{\sqrt{R_{XX}(0)}} \pm \frac{Y(t+\tau)}{\sqrt{R_{YY}(0)}}]^2 = \geq 0$

$E[\frac{X^2(t)}{\sqrt{R_{XX}(0)}} + \frac{Y^2((t+\tau)}{\sqrt{R_{YY}(0)}}] \pm 2\ \frac{X(t)Y(t+\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}}]\ \geq 0$

$E[\frac{X^2(t)}{\sqrt{R_{XX}(0)}}] + E[\frac{Y^2((t+\tau)}{\sqrt{R_{YY}(0)}}] \pm 2\ E[\frac{X(t)Y(t+\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}}]\ \geq 0$

We know that $E[X^2(t)] = R_{XX}(0)$ and $E[Y^2(t)] = R_{YY}(0)$ and $E[X(t)\ X(t+\tau)] = R_{XY}(\tau)$

Therefore $\frac{R_{XX}(0)}{R_{XX}(0)} + \frac{R_{YY}(0)}{R_{YY}(0)} \pm\ 2\frac{R_{XY}(\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}} \geq 0$

$2 \pm 2\frac{R_{XY}(\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}}\ \geq 0$

$1 \pm\ \frac{R_{XY}(\tau)}{\sqrt{R_{XX}(0)R_{YY}(0)}}\ \geq 0$

$\sqrt{R_{XX}(0)R_{YY}(0)}. \geq |R_{XY}(\tau)|$ Or

$|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$ hence proved.

Hence the absolute value of the cross correlation function is always less than or equal to the geometrical mean of the autocorrelation functions.

3. If $R_{XX}(\tau)$ and $R_{YY}(\tau)$ are the autocorrelation functions of X(t) and Y(t) respectively then the cross correlation satisfies the inequality: $|R_{XY}(\tau)| \leq \frac{1}{2}[R_{XX}(0)+ R_{YY}(0)]$.
Proof: We know that the geometric mean of any two positive numbers cannot exceed their arithmetic mean that is if $R_{XX}(\tau)$ and $R_{YY}(\tau)$ are two positive quantities then at τ=0,
$\sqrt{R_{XX}(0)R_{YY}(0)}\ \leq \frac{1}{2}[R_{XX}(0)+ R_{YY}(0)]$.We know that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$

Therefore $|R_{XY}(\tau)| \leq \frac{1}{2}[R_{XX}(0)+ R_{YY}(0)]$. Hence proved.

4. If two random processes X(t) and Y(t) are statistically independent and are at least WSS, then $R_{XY}(\tau) = \bar{X}\ \bar{Y}$.
Proof: Let two random processes X(t) and Y(t) be jointly WSS, then we know that

$R_{XY}(\tau) = E[X(t)\ Y(t+\tau)]$ Since X(t) and Y(t) are independent

$R_{XY}(\tau) = E[X(t)]E[\ Y(t+\tau)]$

Therefore $R_{XY}(\tau) = \bar{X}\,\bar{Y}$ hence proved.

5. If two random processes X(t) and Y(t) have zero mean and are jointly WSS, then $\lim_{|\tau|\to\infty} R_{XY}(\tau) = 0$.

   Proof: We know that $R_{XY}(\tau) = E[X(t)\,Y(t+\tau)]$. Taking the limits on both sides

   $\lim_{|\tau|\to\infty} R_{XY}(\tau) = \lim_{|\tau|\to\infty} E[X(t)\,Y(t+\tau)]$.

   As $|\tau| \to \infty$, the random processes X(t) and Y(t) can be considered as independent processes therefore

   $\lim_{|\tau|\to\infty} R_{XY}(\tau) = E[X(t)]E[Y(t+\tau)] = \bar{X}\,\bar{Y}$

   Given $\bar{X} = \bar{Y} = 0$

   Therefore $\lim_{|\tau|\to\infty} R_{XY}(\tau) = 0$. Similarly $\lim_{|\tau|\to\infty} R_{YX}(\tau) = 0$. Hence proved.

MULTIPLE CHOICE QUESTIONS

1. Let x(t) is a random process which is wide sense stationary, then

A) E[x(t)]=constant B) E[x(t) x(t+T)]=Rxx(T)

C) E[x(t)]= constant and E[x(t) x(t+T)]=Rxx(T) D) $E[x^2(t)]=0$

2. The PDF $f_X(x)$ is defined as

A) Integral of CDF B) Derivative of CDF

C) Equal to CDF D) Partial derivative of CDF

3. Let S_1 and S_2 be the sample spaces of two sub experiments. The combined sample space S is given by

A) S_1 X S_2 B) $S_1 - S_2$

C) $S_1 + S_2$ D) $S_1 | S_2$

4. The relation between conditional probabilities P (A|B) and P (B|A) is derived using one of the following theorems

A) Bernoulli's B) Maxwell's

C) De Moivre D) Bayes

5. The value of $F_X(-\infty)$ is

A) ∞ B) 1

C) 0.5 D) 0

6. Probability density function of the sum of a large no. of random variables approaches

A) Rayleigh distribution B) Uniform distribution

C) Gaussian distribution D) Poisson distribution

7. A mixed random variable is one having

A) Discrete values only B) - ∞ to 0 only

C) Both continuous and discrete D) Continuous values only

8. For an ergodic process

A) Mean is necessarily zero B) Mean square value infinity

C) All time averages are zer0 D) Mean square value is independent of time

9. The moment generating function of X, $M_X(v)$ is expressed as

A) $E[e^v]$ B) $E[e^{vx}]$

C) $e^v x$ D) $E(e^{2x}$

10. The joint probability density function is defined as

A) Derivative of the joint pdf B) Second derivative of the joint pdf

C) Sum of two individual pdfs D) Integration of the joint pdf

Key

1. D
2. B
3. A
4. D
5. D
6. C
7. C
8. B

9. B

10. B

FILL IN THE BLANKS

1. The characteristic function $\phi x(\omega)$ at $\omega = 0$ is _____

2. The normalized third central moment is known as___________

3. If a continuous random variable X has the probability density function f(x)=3/2(1-x2), 0<x<1, then the mean of X is ______

4. If the probability density function of a random variable X is f(x) = kx(x-1) in $1\leq x\leq 4$ and p($1\leq x\leq 3$) =1/3, the value of k is _________

5. For N random variables, the sum YN = X1+X2+….XN, has Gaussian random variable as N tends to ___________

6. For mutually exclusive events, the joint probability is _______

7. The conditional probability for two events can be denoted as ______

8. If FX,Y(∞,Y)=FY(y), it is a ______________function

9. Let A be any event defined on a sample space, the P(A) is _______

10. Central limiting theorem is mostly applicable to statistically ____________

Key:

1	1
2	Skewness of the density function
3	3/8
4	1/14
5	Infinity
6	Zero
7	P (A|B)
8	Marginal distribution
9	≥0
10	Independent random variables

UNIT-V

2 MARKS QUESTIONS WITH ANSWERS

1. **Define the Power Spectral Density of a random Process?**

Ans:

Let

$$X_T(t) = X(t) \quad -T < t < T$$
$$= 0 \quad \text{otherwise}$$
$$= X(t) rect(\frac{t}{2T})$$

where $rect(\frac{t}{2T})$ is the unity-amplitude rectangular pulse of width $2T$ cantered at origin. As $t \to \infty$, $X_T(t)$ will represent the random process $X(t)$.

2. Define the cross power spectral density?

Ans:

Given two real jointly WSS random processes $\{X(t)\}$ and $\{Y(t)\}$ the cross power spectral density (CPSD) $S_{XY}(\omega)$ is defined as

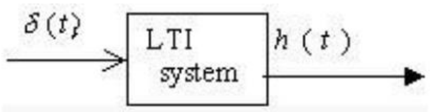
$$\boxed{S_{XY}(\omega) = \lim_{T\to\infty} E \frac{FTX_T^*(\omega) FTY_T(\omega)}{2T}}$$

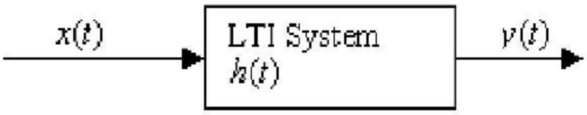
Where $FTX_T(\omega)$ and $FTY_T(\omega)$ are the Fourier transform of the truncated processes $X_T(t) = X(t)rect(\frac{t}{2T})$ and $Y_T(t) = Y(t)rect(\frac{t}{2T})$ respectively and *denotes the complex conjugate operation.

3. Write the response of LTI system to deterministic input?

Ans:

As shown in Figure 2, a linear system can be characterised by its impulse response $h(t) = T\delta(t)$ where $\delta(t)$ is the *Dirac delta function.*





$$y(t) = x(t) * h(t) = h(t) * x(t)$$

4. Write the relation Between Power-spectral Density and Autocorrelation function?

Ans:

Power-spectral Density and Autocorrelation function form the Fourier transform pair given by and this is to referred as wiener-khintchine relation.

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau}\, d\tau$$

$$R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega)\, e^{j\omega\tau} . d\omega$$

5. Write a short note on power density spectrum of response?

Ans:

Consider the random process X(t) is applied to a LTI system having a transfer function H(w). the output response Y(t) , if the power spectrum of the input process is $S_{xx}(w)$ then the power spectrum of the output response is given by

$$S_{yy}(w) = |H(W)|^2\, S_{xx}(w)$$

3 MARKS QUESTIONS WITH ANSWERS

1. Explain the importance of the LTI System?

Ans:

In many applications, physical systems are modeled as linear time-invariant (LTI) systems. The dynamic behavior of an LTI system to deterministic inputs is described by linear differential equations. We are familiar with time and transform domain (such as Laplace transform and Fourier transform) techniques to solve these differential equations. In this lecture, we develop the technique to analyze the response of an LTI system to WSS random process.

The purpose of this study is two-folds:

- Analysis of the response of a system
- Finding an LTI system that can optimally estimate an unobserved random process from an observed process. The observed random process is statistically related to the unobserved random process. For example, we may have to find LTI system (also called a filter) to estimate the signal from the noisy observations.

2. List properties of the Power Density Spectrum ?

Ans:

(1) $S_{XY}(-\omega) = S_{XY}(\omega)$ and $S_{YX}(-\omega) = S_{YX}(\omega)$

(2) The real part of $S_{XY}(\omega)$ and real part $S_{YX}(\omega)$ are even functions of ω i.e. Re $[S_{XY}(\omega)]$ and Re $[S_{YX}(\omega)]$ are even functions.

(3) The imaginary part of $S_{XY}(\omega)$ and imaginary part $S_{YX}(\omega)$ are odd functions of ω i.e. Im $[S_{XY}(\omega)]$ and Im $[S_{YX}(\omega)]$ are odd functions.

(4) $S_{XY}(\omega)=0$ and $S_{YX}(\omega)=0$ if X(t) and Y(t) are Orthogonal.

3. Define power density spectrum

Ans: <u>Power Density Spectrum:</u> The power spectrum of a WSS random process X (t) is defined as the Fourier transform of the autocorrelation function $R_{XX}(\tau)$ of X (t). It can be expressed as

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$$

We can obtain the autocorrelation function from the power spectral density by taking the inverse Fourier transform i.e

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega\tau}\, d\omega$$

Therefore, the power density spectrum $S_{XX}(\omega)$ and the autocorrelation function $R_{XX}(\tau)$ are Fourier transform pairs.

The power spectral density can also be defined as

$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_{T(\omega)}|^2]}{2T}$$

Where $X_T(\omega)$ is a Fourier transform of X(t) in interval [-T,T]

4. Difficulty in Fourier Representation of a Random Process

Ans :1.The Fourier transform of a WSS process $X(t)$ can not be defined by the integral

$$F[X(t)] = \int_{-\infty}^{\infty} X(t).e^{-j\omega t}.dt$$

2. the existence of the above integral would have implied the existence the Fourier transform of every realization of $X(t)$.

3. But the very notion of stationarity demands that the realization does not decay with time and the first condition of Dirichlet is violated

4. This difficulty is avoided by a frequency-domain representation of $X(t)$ interms of the *power spectral density (PSD).*

5. The power of a WSS process $X(t)$ is a constant and given by .

The PSD denotes the distribution of this power over frequencies

5. The autocorrelation function of a WSS process $\{X(t)\}$ is given by

$$R_X(\tau) = a^2 e^{-b|\tau|} \quad b > 0$$

Find the power spectral density of the process?

Ans:

$$S_X(\omega) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau} d\tau$$

$$= \int_{-\infty}^{\infty} a^2 e^{-b|\tau|} e^{-j\omega\tau} d\tau$$

$$= \int_{-\infty}^{0} a^2 e^{b\tau} e^{-j\omega\tau} d\tau + \int_{0}^{\infty} a^2 e^{-b\tau} e^{-j\omega\tau} d\tau$$

$$= \frac{a^2}{b - j\omega} + \frac{a^2}{b + j\omega}$$

$$= \frac{2a^2 b}{b^2 + \omega^2}$$

5 MARKS QUESTIONS WITH ANSWERS

1. Explain the Power Density Spectrum and its properties?

Ans:

In this unit we will study the characteristics of random processes regarding correlation and covariance functions which are defined in time domain. This unit explores the important concept of characterizing random processes in the frequency domain. These characteristics are called spectral characteristics. All the concepts in this unit can be easily learnt from the theory of Fourier transforms.

Consider a random process X (t). The amplitude of the random process, when it varies randomly with time, does not satisfy Dirichlet's conditions. Therefore it is not possible to apply the Fourier transform directly on the random process for a frequency domain analysis. Thus the autocorrelation function of a WSS random process is used to study spectral characteristics such as power density spectrum or power spectral density (psd).

<u>**Power Density Spectrum:**</u> The power spectrum of a WSS random process X (t) is defined as the Fourier transform of the autocorrelation function $R_{XX}(\tau)$ of X (t). It can be expressed as

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$$

We can obtain the autocorrelation function from the power spectral density by taking the inverse Fourier transform i.e

$$R_{XX}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega)\, e^{j\omega\tau}\, d\omega$$

Therefore, the power density spectrum $S_{XX}(\omega)$ and the autocorrelation function $R_{XX}(\tau)$ are Fourier transform pairs.

The power spectral density can also be defined as

$$S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_{T(\omega)}|^2]}{2T}$$

Where $X_T(\omega)$ is a Fourier transform of X(t) in interval [-T,T]

Average power of the random process: The average power P_{XX} of a WSS random process X(t) is defined as the time average of its second order moment or autocorrelation function at τ =0.

Mathematically, $P_{XX} = A\{E[X^2(t)]\}$

$$P_{XX} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} E[X^2(t)]dt$$

Or $P_{XX} = R_{XX}(\tau)|\tau = 0$

We know that from the power density spectrum,

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX(\omega)}\, e^{j\omega\tau}\, d\omega$$

At τ =0 $P_{XX} = R_{XX}(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX(\omega)}\, d\omega$

Therefore average power of X(t) is

Properties of power density spectrum: The properties of the power density spectrum $S_{XX}(\omega)$ for a WSS random process X(t) are given as

(1) $S_{XX}(\omega) \geq 0$

Proof: From the definition, the expected value of a non negative function $E[\left|\left|X_{T(\omega)}\right|^2\right|]$ is always non-negative.

Therefore $S_{XX}(\omega) \geq 0$ hence proved.

(2) The power spectral density at zero frequency is equal to the area under the curve of the autocorrelation $R_{XX}(\tau)$ i.e. $S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX(\tau)}\, d\tau$

Proof: From the definition we know that $S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX(\tau)}\, e^{-j\omega\tau}\, d\tau$ at $\omega=0$,

$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX(\tau)}\, d\tau$ hence proved

(3) The power density spectrum of a real process X(t) is an even function i.e.

$S_{XX}(-\omega) = S_{XX}(\omega)$

Proof: Consider a WSS real process X(t). then

$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{-j\omega\tau}\, d\tau$ also $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$

Substitute $\tau = -\tau$ then

$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau)\, e^{-j\omega\tau}\, d\tau$

Since X(t) is real, from the properties of autocorrelation we know that, $R_{XX}(-\tau) = R_{XX}(\tau)$

Therefore $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, e^{j\omega\tau}\, d\tau$

$S_{XX}(-\omega) = S_{XX}(\omega)$ hence proved.

(4) $S_{XX}(\omega)$ is always a real function

Proof: We know that $S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$

Since the function $\left||X_T(\omega)|^2\right|$ is a real function, $S_{XX}(\omega)$ is always a real function hence proved.

(5) If $S_{XX}(\omega)$ is a psd of the WSS random process X(t), then

$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega = A\{E[X^2(t)]\} = R_{XX}(0)$ or The time average of the mean square value of a WSS random process equals the area under the curve of the power spectral density.

Proof: We know that $R_{XX}(\tau) = A\{E[X(t+\tau)X(t)]\}$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega$ at $\tau = 0$,

$R_{XX}(0) = A\{E[X^2(t)]\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\, d\omega$ = Area under the curve of the power spectral density. Hence proved.

(6) If X(t) is a WSS random process with psd $S_{XX}(\omega)$, then the psd of the derivative of X(t) is equal to ω^2 times the psd $S_{XX}(\omega)$. That is $S\dot{X}\dot{X}(\omega) = \omega^2 S_{XX}(\omega)$.

Proof: We know that $S_{XX}(\omega) = \lim_{T\to\infty} \frac{E[|X_{T(\omega)}|^2]}{2T}$ and $X_T(\omega) = \int_{-T}^{T} X(t)e^{-j\omega t}dt$

$= \dot{X}_T(\omega) = \frac{d\,X_T(\omega)}{dt} = \frac{d}{dt}\int_{-T}^{T} X(t)e^{-j\omega t}dt$

$= \int_{-T}^{T} X(t)\frac{d}{dt}\, e^{-j\omega t}dt$

$= \int_{-T}^{T} X(t)(-j\omega)\, e^{-j\omega t}dt$

$=(-j\omega)\int_{-T}^{T} X(t)\, e^{-j\omega t}dt$

Therefore $S\dot{X}\dot{X}(\omega) = \lim_{T\to\infty} \frac{E[|\dot{X}_T(\omega)|^2]}{2T}$

$= \lim_{T\to\infty} \frac{E[|(-J\omega)X_{T(\omega)}|^2]}{2T}$

$= \lim_{T\to\infty} \frac{\omega^2 E[|(-J\omega)X_{T(\omega)}|^2]}{2T}$

$S\dot{X}\dot{X}(\omega) = \omega^2 \lim_{T\to\infty} \frac{E[|\dot{X}_T(\omega)|^2]}{2T}$

Therefore $S\dot{X}\dot{X}(\omega) = \omega^2 S_{XX}(\omega)$ hence proved.

2. Explain the Power Density Spectrum and its properties?

Ans: **Cross power density spectrum:** Consider two real random processes X(t) and Y(t). which are jointly WSS random processes, then the cross power density spectrum is defined as the Fourier transform of the cross correlation function of X(t) and Y(t).and is expressed as

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$ and $S_{YX}(\omega) = \int_{-\infty}^{\infty} R_{YX}(\tau)\, e^{-j\omega\tau}\, d\tau$ by inverse Fourier transformation, we can obtain the cross correlation functions as

$R_{XY}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega)\, e^{j\omega\tau}\, d\omega$ and $R_{YX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{YX}(\omega)\, e^{j\omega\tau}\, d\omega$

Therefore the cross psd and cross correlation functions are forms a Fourier transform pair.

If $X_T(\omega)$ and $Y_T(\omega)$ are Fourier transforms of X(t) and Y(t) respectively in interval [-T,T], Then the cross power density spectrum is defined as

$S_{XY}(\omega) = \lim_{T\to\infty} \frac{E[\left|X_T(\omega)Y_T(\omega)\right|^1]}{2T}$ and $S_{YX}(\omega) = \lim_{T\to\infty} \frac{E[\left|Y_T(\omega)X_T(\omega)\right|^1]}{2T}$

Average cross power: The average cross power P_{XY} of the WSS random processes X(t) and Y(t) is defined as the cross correlation function at τ =0. That is

$P_{XY} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} R_{XY}(t,t)dt$ or

$P_{XY} = R_{XY}(\tau)|\tau = 0 = R_{XY}(0)$ Also $P_{XY} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XY}(\omega)\, d\omega$ and $P_{YX} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{YX}(\omega)\, d\omega$

Properties of cross power density spectrum: The properties of the cross power for real random processes X(t) and Y(t) are given by

(1) $S_{XY}(-\omega) = S_{XY}(\omega)$ and $S_{YX}(-\omega) = S_{YX}(\omega)$

Proof: Consider the cross correlation function $R_{XY}(\tau)$. The cross power density spectrum is $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$

Let $\tau = -\tau$ Then

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(-\tau)\, e^{j\omega\tau}\, d\tau$ Since $R_{XY}(-\tau) = R_{XY}(\tau)$

$S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j-\omega\tau}\, d\tau$

Therefore $S_{XY}(-\omega) = S_{XY}(\omega)$ Similarly $S_{YX}(-\omega) = S_{YX}(\omega)$ hence proved.

(2) The real part of $S_{XY}(\omega)$ and real part $S_{YX}(\omega)$ are even functions of ω i.e. Re $[S_{XY}(\omega)]$ and Re $[S_{YX}(\omega)]$ are even functions.

Proof: We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$ and also we know that

$e^{-j\omega\tau}$=cosωt-jsinωt, Re $[S_{XY}(\omega)] = \int_{-\infty}^{\infty} R_{XY}(-\tau) \cos\omega t\, d\tau$

Since cos ωt is an even function i.e. cos ωt = cos (-ωt)

Re $[S_{XY}(\omega)] = \int_{-\infty}^{\infty} R_{XY}(\tau) \cos\omega t\, d\tau = \int_{-\infty}^{\infty} R_{XY}(\tau) \cos(-\omega t)\, d\tau$

Therefore $S_{XY}(\omega) = S_{XY}(-\omega)$ Similarly $S_{YX}(\omega) = S_{YX}(-\omega)$ hence proved.

(3) The imaginary part of $S_{XY}(\omega)$ and imaginary part $S_{YX}(\omega)$ are odd functions of ω i.e. Im $[S_{XY}(\omega)]$ and Im $[S_{YX}(\omega)]$ are odd functions.

Proof: We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$ and also we know that

$e^{-j\omega\tau}$=cosωt-jsinωt,

Im $[S_{XY}(\omega)] = \int_{-\infty}^{\infty} R_{XY}(\tau)(-\sin\omega t)\, d\tau = -\int_{-\infty}^{\infty} R_{XY}(\tau) \sin\omega t\, d\tau = -$ Im $[S_{XY}(\omega)]$

Therefore Im $[S_{XY}(\omega)] = -$ Im $[S_{XY}(\omega)]$ Similarly Im $[S_{YX}(\omega)] = -$ Im $[S_{YX}(\omega)]$ hence proved.

(4) $S_{XY}(\omega)=0$ and $S_{YX}(\omega)=0$ if X(t) and Y(t) are Orthogonal.

Proof: From the properties of cross correlation function, We know that the random processes X(t) and Y(t) are said to be orthogonal if their cross correlation function is zero.

i.e. $R_{XY}(\tau) = R_{YX}(\tau) = 0$.

We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$

Therefore $S_{XY}(\omega)=0$. Similarly $S_{YX}(\omega)=0$ hence proved.

(5) If X(t) and Y(t) are uncorrelated and have mean values $\bar{X}$ and $\bar{Y}$, then

$S_{XY}(\omega)=2\pi\bar{X}\,\bar{Y}\,\delta(\omega)$.

Proof: We know that $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)\, e^{-j\omega\tau}\, d\tau$

$= S_{XY}(\omega) = \int_{-\infty}^{\infty} E[X(t)Y(t+\tau)]e^{-j\omega\tau}\, d\tau$

Since X(t) and Y(t) are uncorrelated, we know that

$E[X(t)Y(t+\tau)= E[X(t)]E[Y(t+\tau)]$

Therefore $S_{XY}(\omega) = \int_{-\infty}^{\infty} E[X(t)]E[Y(t+\tau)]e^{-j\omega\tau}\, d\tau$

$S_{XY}(\omega) = \int_{-\infty}^{\infty} \bar{X}\bar{Y}e^{-j\omega\tau}\, d\tau$

$S_{XY}(\omega) =\bar{X}\,\bar{Y}\int_{-\infty}^{\infty} e^{-j\omega\tau}\, d\tau$

Therefore $S_{XY}(\omega)=2\pi\bar{X}\,\bar{Y}\,\delta(\omega)$. hence proved.

3. Difficulty in Fourier Representation of a Random Process

Ans: To design any LTI filter which is intended to extract or suppress the signal, it is necessary to understand how the strength of a signal is distributed in the frequency domain, relative to the strengths of other ambient signals. Similar to the deterministic signals, it turns out to be just as true in the case of random signals.

There are two immediate challenges in trying to find an appropriate frequency-domain description for a WSS random process. First, individual sample functions typically don’t have transforms that are ordinary, well-behaved functions of frequency; rather, their transforms are only defined in the sense of generalized functions. Second, since the particular sample function is determined as the outcome of a probabilistic experiment, its features will actually be random, and it is to be searched for features of the transforms that are representative of the whole class of sample functions, i.e., of the random process as a whole.

The present module focuses on the expected power in the signal which is a measure of signal strength and will be shown that it meshes nicely with the second moment characterizations of a WSS process. For a process that is second-order ergodic, this will also correspond to the time average power in any realization.

Description:

1) Ideally, all the sample functions of a random process are assumed to exist over the entire time interval $(-\infty, +\infty)$, and thus, are power signals.Thus, the existence of Power spectral density should be enquired.

2) The concept of Power spectral density may not appear to be meaningful for a random process for the reasons as follows:

3) It may not be possible to describe a sample function analytically
4) For a given process, every sample function may be different from another one

5) Hence, even PSD exists for each sample function, it may be different for different sample functions
6) It is possible to define a meaningful PSD for a stationary(at least in the wide sense) random process.
7) For non-stationary processes, PSD does not exist.

8) For random signals and random Variables, because of the anon availability of the enough information to predict the output with certainty, the respective measures are done in-terms of averages.

9) On these lines, the PSD of a random process is defined as a weighted mean of the PSDs of all sample functions, as it is not known exactly which of the sample functions may occur in a given trial.

4. Explain Difficulty in Fourier Representation of a Random Process

Ans: 1.The Fourier transform of a WSS process $X(t)$ can not be defined by the integral

$$F[X(t)] = \int_{-\infty}^{\infty} X(t).e^{-j\omega t}\ .dt$$

2. the existence of the above integral would have implied the existence the Fourier transform of every realization of $X(t)$.

3. But the very notion of stationarity demands that the realization does not decay with time and the first condition of Dirichlet is violated

4. This difficulty is avoided by a frequency-domain representation of $X(t)$ in

terms of the *power spectral density (PSD)*.

5. The power of a WSS process $X(t)$ is a constant and given by ^{2} .

The PSD denotes the distribution of this power over frequencies.

Defining the Power Spectral Density of a random Process

Let

$$X_T(t) = X(t) \qquad -T < t < T$$
$$= 0 \qquad \text{otherwise}$$
$$= X(t)\,rect\left(\frac{t}{2T}\right)$$

where $rect\left(\frac{t}{2T}\right)$ is the unity-amplitude rectangular pulse of width $2T$ cantered at origin. As $t \to \infty$, $X_T(t)$ will represent the random process $X(t)$.

Define $F[X_T(t)] = X_T(\omega) = \int_{-\infty}^{\infty} X_T(t).e^{-j\omega t}.dt$

Applying Parseval's theorem to find the energy of the signal

$$\int_{-T}^{T} {X_T}^2(t).dt = \int_{-\infty}^{\infty} |X_T(\omega)|^2.d\omega$$

Therefore, the power associated with $X_T(t)$ is

$$\frac{1}{2T}\int_{-T}^{T} {X_T}^2(t).dt = \frac{1}{2T}\int_{-\infty}^{\infty} |X_T(\omega)|^2.d\omega$$

The average power is given by

$$\frac{1}{2T}E\left[\int_{-T}^{T} {X_T}^2(t).dt\right] = \frac{1}{2T}E\left[\int_{-\infty}^{\infty} |X_T(\omega)|^2.d\omega\right] = E\left[\int_{-\infty}^{\infty} \frac{|X_T(\omega)|^2}{2T} d\omega\right]$$

where $E\left[\int_{-\infty}^{\infty} \frac{|X_T(\omega)|^2}{2T}\right]$ is the contribution to the average power at frequency ω and represents the power spectral density for $X_T(t)$. As $T \to \infty$, the left-hand side in the above expression represents the average power of $X(t)$. Therefore, the PSD $S_x(\omega)$ of the process $X(t)$ is defined in the limiting sense by

$$S_{xx}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$$

Thus, The PSD $S_x(\omega)$ of a random process X(t) is defined as the ensemble average of the PSDs of all sample functions Thus,

$$S_{xx}(\omega) = \lim_{T\to\infty}\left[\frac{\overline{|X_T(\omega)|^2}}{T}\right]$$

Where, $X_T(\omega)$ is theFourier Transform of the truncated random process $X(t).rect(\frac{t}{T})$ and the bar represents the ensemble average.
The ensemble averaging is done before the limiting operation.

5. Determine the relation Between Power-spectral Density and Autocorrelation function of the Random Process

Ans:

We have PSD$S_{xx}(\omega) = \lim_{T\to\infty} \frac{E[|X_T(\omega)|^2]}{2T}$

$$= \lim_{T\to\infty} \frac{E[X^*{}_T(\omega)X_T(\omega).]}{2T}$$

$X_T(\omega) = \int_{-T}^{T} X(t).e^{-j\omega t}.dt$ and $X^*{}_T(\omega) = \int_{-T}^{T} X(t).e^{j\omega t}.dt$

$$S_{xx}(\omega) = \lim_{T\to\infty} \frac{1}{2T} E\left[\int_{-T}^{T} X(t_1).e^{j\omega t_1}.dt_1.\int_{-T}^{T} X(t_2).e^{-j\omega t_2}.dt_2\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} E\left[\int_{-T}^{T}\int_{-T}^{T} X(t_1)\,X(t_2).e^{-j\omega(t_2-t_1)}.dt_1 dt_2.\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T}\int_{-T}^{T} E[X(t_1)\,X(t_2)].e^{-j\omega(t_2-t_1)}.dt_1 dt_2$$

$$=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T}R_{xx}(t_1,t_2)\,e^{-j\omega(t_2-t_1)}\,.\,dt_1dt_2$$

Consider the inverse Fourier Transform of PSD i.e. $\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{xx}(\omega)\,e^{j\omega\tau}\,.\,d\omega$

$F^{-1}[S_{xx}(\omega)]=$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}[\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T}R_{xx}(t_1,t_2)\,e^{-j\omega\,(t_2-t_1)}\,.\,dt_1dt_2]\,e^{j\omega\tau}\,.\,d\omega$$

$$=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T}R_{xx}(t_1,t_2)\,\frac{1}{2\pi}.\int_{-\infty}^{\infty}e^{-j\omega\,(t_2-t_1)}\,.\,d\omega.\,dt_1dt_2$$

Since, $F[\delta(t)]=1,\to\frac{1}{2\pi}\int_{-\infty}^{\infty}1.\,e^{j\omega t}\,d\omega=\delta(t)$

On similar lines, $\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{j\omega\,(\tau-t_2+t_1)}\,d\omega=\delta(\tau-t_2+t_1)$

$$F^{-1}[S_{xx}(\omega)]=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\int_{-T}^{T}R_{xx}(t_1,t_2)\delta(\tau-t_2+t_1).\,dt_1dt_2$$

since $\delta(\tau-t_2+t_1)=1\ at\ \tau-t_2+t_1=0\ i.e.\ t_2=\tau+t_1$

$$F^{-1}[S_{xx}(\omega)]=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}R_{xx}(t_1,\tau+t_1)\,dt_1$$

Let $t_1=\tau\to dt_1=d\tau$

Hence, $F^{-1}[S_{xx}(\omega)]=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}R_{xx}(t,t+\tau)\,dt$

The RHS of the above eq. is the time average of Auto correlation function.

Thus, Time average of Autocorrelation function and the PSD form a Fourier Transform Pair.

MULTIPLE CHOICE QUESTIONS

1. The signal x(t) = A cos (ω0t + φ) is

A. energy signal B. power signal

C. energy Power D. none

2. An energy signal has G(*f*) = 10. Its energy density spectrum is

A. 10

B. 100

C. 50

D. 20

3. The spectral density of white noise is

A. Exponential B. Uniform

C. Poisson D. Gaussian

4. The area under Gaussian pulse $\int_{-\infty}^{\infty} e^{-\pi t^2} dt$ is

A. Unity B. Infinity

C .Pulse D. Zero

5. The power spectral density of WSS IS always

A. Negative B. Non- Negative

C .Positive D. Can be Positive or negative

6. Convolution is used to find

A Amount of similarity between the signals B Response of the system

C Multiplication of the signals D Fourier transform

7. The Fourier transform of a rectangular pulse is

A another rectangle pulse B Square pulse

C Triangular pulse D Sinc pulse

8. The probability cumulative distribution function must be monotone and

A increasing B decreasing

C non-increasing D non-decreasing

9 if X(t) and Y(t) are orthogonal, then

A $S_{xy}(w)=0$ B $S_{xy}(w)=1$

C $S_{xy}(w)>1$ D $S_{xy}(w)<1$

10. When two honest coins are simultaneously tossed, the probability of two heads on any given trial is:

A 1 B 2

C 1/2 D 1/4

Key

1. B

2. B

3. B

4. A

5. B

6. B

7. D

8. D

9. A

10. D

FILL IN THE BLANKS

1. For the probability density function of a random variable X given by $f_x(x) = 5e^{-Kx}u(x)$ where u(x) is the unit step function, the value of K is_____

2. Discrete time system is stable if the poles are_____

3. Convolution is used to find_________

4. The autocorrelation of a sinusoid is_______

5. The probability cumulative distribution function must be monotone and____

6. The time average of the autocorrelation function and power spectral density form a pair of__

7. The power spectral density of WSS is always_____________

8. For a WSS process, psd at zero frequency gives___________

9. The cross spectral density $S_{yx}(w)$=__________________

10. The cross power density of X(t) and Y(t) can be obtained by $S_{yx}(w)$=_______

Key:

1	5
2	within unit circle
3	response of the system
4	Sinusoid
5	Non Decreasing
6	Fourier transform
7	Non negative
8	Area under the graph of a power spectral density
9	$S_{xx}(-w)$
10	$H^*(w)\ S_{xx}(w)$

17) CO's, PO's mapping

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2		1								
CO2	2	3		1	1							
CO3	3	2		2	2							
CO4	2	1		2								
CO5	2	1		2								

Legends: 1 – Low

2 – Medium

3 – High

18) Beyond Syllabus Topics With Material

1. Filter design

In many applications of signal processing we want to change the relative ampli-tudes and frequency contents of a signal. This process is generally referred to as filtering. Since the Fourier transform of the output is product of input Fourier transform and frequency response of the system, we have to use appropriate frequency response.

Ideal frequency selective filters:

An ideal frequency reflective filter passes complex exponential signal. for a given set of frequencies and completely rejects the others. ideal low pass filter (LPF), ideal high pass filter (HPF), ideal band pass filter (BPF) and ideal backstop filter (BSF).

The ideal filters have a frequency response that is real and non-negative, in other words, has a zero phase characteristics. A linear phase characteristics introduces a time shift and this causes no distortion in the shape of the signal in the passband.

Since the Fourier transfer of a stable impulse response is continuous function of ω, can not get a stable ideal filter.

Filter specification:

Since the frequency response of the realizable filter should be a continuous func-tion, the magnitude response of a lowpass filter is specified with some acceptable tolerance. Moreover, a transition band is specified between the passband and stop band to permit the magnitude to drop off smoothly.

In the passband magnitude the frequency response is within $\pm\delta_p$ of unity

$$(1 - \delta_p) \le |H(e^{j\omega})| \le (1 + \delta_p), \quad |\omega| \le \omega_p$$

In the stopband

$$|H(e^{j\omega})| \le \delta_s, \quad |\omega| < \omega_s \le \pi$$

The frequencies ω_p and ω_s are respectively, called the passband edge frequency and the stopband edge frequency. The limits on tolerances δ_p and δ_s are called the peak ripple value. Often the specifications of digital filter are given in terms of the loss function $G(w) = -20log_{10}|H(e^{j\omega})|$, in dB. The loss specification of digital filter are

$$\alpha_p = -20log_{10}(1 - \delta_p)dB$$

$$\alpha_s = -20log_{10}\delta_s dB$$

Sometimes the maximum value in the passband is assumed to be unity and the maximum pass band deviation, denoted as $1/\sqrt{1+E2}$ is given the minimum value of the magnitude in passband. The maximum stopband magnitude is denoted by $1/A$. The quantity αmax is given by
$\alpha max = 20log10(1 + E2)^{1/2}dB$

2. AWG noise characteristics

Additive white Gaussian noise (AWGN) is a basic noise model used in Information theory to mimic the effect of many random processes that occur in nature. The modifiers denote specific characteristics:

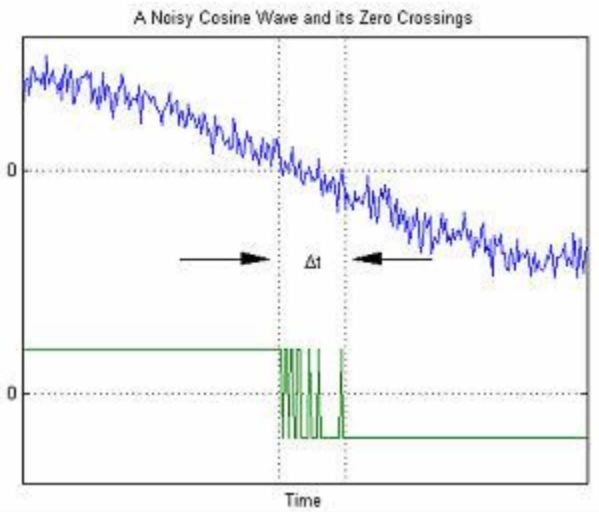
- Additive because it is added to any noise that might be intrinsic to the information system.
- White refers to the idea that it has uniform power across the frequency band for the information system. It is an analogy to the color white which has uniform emissions at all frequencies in the visible spectrum.
- Gaussian because it has a normal distribution in the time domain with an average time domain value of zero.

Wideband noise comes from many natural sources, such as the thermal vibrations of atoms in conductors (referred to as thermal noise or Johnson-Nyquist noise), shot noise, black body radiation from the earth and other warm objects, and from celestial sources such as the Sun. The central limit theorem of probability theory indicates that the summation of many random processes will tend to have distribution called Gaussian or Normal.

AWGN is often used as a channel model in which the only impairment to communication is a linear addition of wideband or white noise with a constant spectral density(expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. The model does not account for fading, frequency selectivity, interference, nonlinearityor dispersion. However, it produces simple and tractable mathematical models which are useful for gaining insight into the underlying behavior of a system before these other phenomena are considered.

The AWGN channel is a good model for many satellite and deep space communication links. It is not a good model for most terrestrial links because of multipath, terrain blocking, interference, etc. However, for terrestrial path modeling, AWGN is commonly used to simulate background noise of the channel under study, in addition to multipath, terrain blocking, interference, ground clutter and self interference that modern radio systems encounter in terrestrial operation.

Effects in time domain



Zero-Crossings of a Noisy Cosine

In serial data communications, the AWGN mathematical model is used to model the timing error caused by random jitter (RJ).

The graph to the right shows an example of timing errors associated with AWGN. The variable Δt represents the uncertainty in the zero crossing. As the amplitude of the AWGN is increased, the signal-to-noise ratio decreases. This results in increased uncertainty Δt.

When affected by AWGN, the average number of either positive going or negative going zero-crossings per second at the output of a narrow band pass filter when the input is a sine wave is:

Where

- f_0 = the center frequency of the filter
- B = the filter bandwidth

- SNR = the signal-to-noise power ratio in linear terms